# Politonomics: A Meta-Theory For Political and Economic Decision Making

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## Abstract

In Social Choice and Individual Values, Kenneth Arrow (1951) said [1], "In a capitalist democracy there are essentially two methods by which social choices can be made: voting, typically used to make 'political' decisions, and the market mechanism, typically used to make 'economic' decisions." He goes on to say, "The methods of voting and the market … are methods of amalgamating the tastes of many individuals in the making of social choices." Initially, Arrow does not distinguish between political and economic systems claiming that both are means of formulating social decisions based on individual input. Arrow then purports to show that there is no rational way to make social decisions based on the amalgamation of individual ones thus ruling out welfare economics or economic democracy and also direct political democracy. The dichotomy between political and economic systems remains with the implication being that representive democracy and capitalist economics are the best systems that can be attained.

Amartya Sen in his Nobel Prize acceptance speech said:

"Arrow's 'impossibility theorem' (formally, the 'General Possibility Theorem') is a result of breathtaking elegance and power, which showed that even some very mild conditions of reasonableness could not be simultaneously satisfied by any social choice procedure, within a very wide family. Only a dictatorship would avoid inconsistencies, but that of course would involve: (1) in politics, an extreme sacrifice of participatory decisions; and (2) in welfare economics, a gross inability to be sensitive to the heterogeneous interests of a diverse population. Two centuries after the flowering of the ambitions of social rationality, in Enlightenment thinking and in the writings of the theorists of the French Revolution, the subject seemed to be inescapably doomed. Social appraisals, welfare economic calculations, and evaluative statistics would have to be, it seemed, inevitably arbitrary or unremediably despotic."

This paper resolves the dichotomy between politics and economics, advances the cause of economic democracy and direct political democracy while calling into question the finality of Arrow's pronouncement that social choice is impossible. We do this by developing a meta-theory from which can be derived methods for both political and economic decision making. Not only does this theory overcome Arrow's Impossibility

Theorem, but it also compensates for strategic voting, an undesirable aspect of decision making according to Gibbard (1973: 41) [2] and Satterthwaite (1975: 10) [3]. Without including the proscription against strategic voting, Arrow's Impossibility Theorem would not make sense or be complete. Arrow's Impossibility Theorem has led to much negativity, even nihilism, among social theorists who think, if Arrow's proclamation is true, there can be no further advancement in societal decision making mechanisms. This gives a theoretical endorsement to capitalist economics and first-past-the-post, majority rule, winner-take-all voting systems.

The politonomics meta-theory spawns both political and economic systems which are indeed possible and which cannot be gamed by strategic voting. In a typical voting system the outcome of an election among several candidates results in one realized outcome – the winner of the election – which applies to all voters. In a typical economic system, a consumer may choose among a variety of possible baskets of consumer items (what Arrow calls "commodity bundles") and work programs with the result that multiple realized outcomes are possible with a unique or quasi-unique outcome for each worker-consumer. As the number of possible realized outcomes of a political-economic decision making process increases, the process becomes more economic and less political in nature and vice versa. We show that, as the number of possible realized outcomes increases, voter/worker-consumer satisfaction or utility increases both individually and collectively. We also derive a method for achieving the best outcome(s) for both the individual and society. Thus Arrow's Impossibility Theorem is rendered obsolete.

This paper gives a theoretical endorsement to the cooperative ownership movement in which enterprises are owned by worker/owner/consumers. Rather than choosing among a number of commodity bundles or consumer baskets which wouldn't be acceptable in any modern economy, a more realistic approach is for worker/owner/consumers to choose among a number of possible work programs and their dollar equivalents in compensation. For example, to get workers to work the night shift, which may be seen as less desirable, it might be necessary to pay a higher hourly rate if the choice among alternatives was made on a democratic basis among the worker/owners.

## Introduction

A possible realized outcome is an alternative or candidate that can apply to or be the choice of a voter or worker/consumer after the voting or selection process occurs. For example, in traditional voting in a single member constituency, there may be several alternatives or candidates, but, after the voting process occurs, there can be only one possible realized outcome and that is the winner of the election. This outcome applies to all voters including the ones that didn't vote for the winning candidate. In practice winner take all voting systems often disenfranchise minorities.

Now imagine a hypothetical political system in which there are two possible realized outcomes. Let's say that after the voting process among several candidates occurs, A and B are both winners and that all those who prefer A to B are governed by A and all those who prefer B to A are governed by B. It's as if there has been a split into two jurisdictions and two constituencies with A governing in one and B governing in the other. Please note that geography need not have anything to do with the two jurisdictions and constituencies.

Alternatively, the voting process can be viewed as being conducted in a double member district as opposed to a single member district in that there are two winners of the election with both representing that district. In a district with a large minority one of the winners would likely represent that minority.

From an economic viewpoint, this process can be conceived of as a group of worker/consumers selecting, out of several possibilities, one or the other of two baskets – A or B – of consumer items or, alternatively, of dollar equivalents and work schedules with associated hourly rates.

In the political case for multiple possible realized outcomes, the winners might all be considered to be members of a parliament so that each voter would be represented to a greater or lesser degree by various members of parliament depending on how closely each member correlated with that voter's expressed preferences. Single member districts give way to multi-member districts.

We take a utilitarian approach but avoid the controversy about **interpersonal comparisons** (Camacho 1982) [4] by assuming that, even for a welfare economy or, the more appropriate term, **economic democracy**, voting methods are used, and hence each individual chooser or voter is allocated the power of one vote thus equalizing all interpersonal comparisons. **Utilitarian** economic methods are used for voting rather than rank order preferences. That is voters *rate* their preferences (using real numbers on some scale) as opposed to *ranking* them (A is preferred to B is preferred to C etc) as Arrow assumes. Hillinger[5] has made the case for utilitarian voting. Lehtinen (2011: 23)[6] asserts: "One reason why one indivdual has one vote under most rules is that each individual's voting choice is considered equally important, and each individual's utility is taken to carry at least roughly equal weight in the welfare function."

All individual choosers can express **preference intensity**, but those preference intensities are considered to have equal weight. We assume, without loss of generality, a method of indicating preferences and intensities consisting of a rating scale of real numbers with the lowest rating being minus one and the highest rating, plus one. A real number between minus one and plus one indicates preference as well as intensity. The chooser's utility or satisfaction is correlated with how high the outcome is on their preference rating scale.

So if candidate A is rated highly on a particular voter's preference rating scale and candidate B is rated quite low, we could say that the member of Congress or Parliament that best represents this voter's interests is candidate A with high intensity. If A is preferred to B, but only slightly, we could say the preference has low intensity. The strategic aspects of a cardinal approach will be dealt with as well so that they are compensated for in the voting process. By using ratings instead of rankings or, technically speaking, a cardinal rather than an ordinal approach, Arrow's impossibility theorem is rendered moot.

## Range/Approval Hybrid Voting for One Outcome

We know that for only one possible realized outcome, there is an optimal voting procedure called range/approval hybrid [7,8] (Smith, Lawrence). Sincere range voting [9] in which each voter expresses their sincere preferences by rating the candidates with a real number between –1 and +1 will tend to maximize social utility. However, this system is subject to strategic voting in which some voters may gain an advantage by expressing their preferences insincerely. Although Lehtinen[5] has shown that strategic voting can actually improve collective or social utility, we take the attitude here that strategic voting is undesirable if done on a random basis giving some voters an advantage over others.

**Strategic considerations** lead to applying a formula for each individual to their sincere preference ratings in order to maximize the outcome for that individual. If no prior information is known regarding other voters' preferences, then all ratings greater than the individual's average are changed to "+1" and all ratings less than the average are changed to "–1". This changes the preference rating to an approval (Brams and Fishburn 1983) [10] style vote which is the best one can do in terms of strategy in the absence of information regarding other voters. This is equivalent to placing a threshold just below the mean of the **preference ratings** and adjusting the ratings of every **sincere preference** above the threshold to "1" and every rating below the threshold to "–1".

Finally, the **approval style votes** for each candidate are summed over all voters and the candidate with the most votes is declared the winner. We will only consider the case in which individual strategy occurs without knowledge of information regarding other voters. The more general case in which knowledge of the sincere preferences of other voters is taken into account before strategizing is beyond the purview of this paper.

The **social decision function** is the part of the process which takes into account the individual preference ratings and outputs the final results. The process of strategizing can be undertaken by individuals on a haphazard basis or it can be done on a universal basis by the social decision function itself just prior to the summation of votes. *If done in this way, it negates the advantage of strategizing by individuals and equalizes the benefits of strategizing for all voters.* There may be a loss or a gain of collective or social utility when strategy is used, but this is either the price to be paid for eliminating strategic advantages for some voters or represents a serendipitous bonus in social utility. Individual utility of the outcome is the individual participant's **sincere preference rating** for that outcome. Social utility of the outcome is the summation over all the individual participants' sincere preference ratings for that outcome.

Therefore, since we know the optimal strategy for each individual, we can take it out of the hands of individuals and make it a part of the social decision function itself. By doing so two things are accomplished: 1) there is nothing to be gained by an individual voter in strategizing their vote so that each voter has an incentive to vote sincerely, and 2) each voter will be assured that they will gain the advantages of an optimal strategy. Thus, as Dowding and Van Hees (Dowding and Van Hees 2007: 38) [11] have stated: "Rather than being concerned about the strategic or gaming nature of voting systems, we should celebrate those aspects … [because] … the possibility of manipulation is often a virtue rather than a vice."

Let m represent the number of possible realized outcomes. In a winner take all district, for example, $m = 1$. When $m > 1$ as in a multi-member district, each voter might identify with the winning candidate who had the highest utility rating on their individual ballot as their personal representative or they might identify with the subset of winning candidates all of whom were above some threshold on their numerical ballot.

In economic terms for $m > 1$, the model suggests that only m worker/consumer baskets (or work schedules with associated compensation rates) will be made available out of a possible $n > m$ and then the worker/consumer will pick one of these, presumably the one with the greatest utility for them. A basket might represent, for example, number of hours worked in return for a certain selection of consumer goods or the dollar equivalent. A cooperative enterprise might best implement such a system using a sophisticated online computer application. There are numerous shift scheduling apps for nurses and others already out there.

## Increasing the Number of Possible Realized Outcomes

Now if we imagine that there are a number of possible realized outcomes, and not just one, we might ask how that affects strategic considerations? Obviously, if there is a very large number of possible realized outcomes compared to the number of voters, there is no need for strategy. Each voter can vote sincerely in the knowledge that they will get their first choice or close to their first choice as an outcome. But let us consider the case of just two possible realized outcomes. Obviously, the threshold can be raised from just below the average of the individual voter's ratings since the voter has two chances to get an outcome closer to their most preferred outcome. However, we want to find the best or optimal placement of the threshold, and we proceed with that in mind.

Let's examine an individual citizen's vote which represents a specification of utilities over the candidates with each utility corresponding to a position on the **preference rating scale** which we choose, without loss of generality, to be between –1 and +1. Each individual voter associates each candidate with a particular utility or real number on that scale. The greater the indicated utility, the higher the probability that a particular candidate will be elected due to that individual alone since utilities for a particular candidate will be added up over all voters. We assume a variable number of possible realized outcomes or alternatives - m. Those outcomes compose the **winning set.** For each m we place the threshold such that the expected individual utility using strategy is a maximum.

The expected value of a discrete random variable is the probability-weighted average of all possible values. In other words, each possible value the random variable can assume is multiplied by its probability of occurring, and the resulting products are summed to produce the expected value. We want the expected value of the set of utilities above threshold which is

$$\mathbf{E}(\mathbf{U}_a) = \sum_{i=1}^{n_a} \mathbf{p}_i \mathbf{u}_i$$

where $U_a$ is the set of utilities above threshold, $n_a$ is the number of utilities above threshold and $p_i$ is the probability of each utility, $u_i$. Let $n_b$ be the number of utilities below threshold so that $n = n_a + n_b$ = total number of utilities. Let $u_a$ be the sum of utilities above threshold and $u_b$ be the sum of utilities below threshold. We assume no knowledge of statistics regarding outcomes of the election process or other voters' preferences. Therefore, the probability of any particular candidate being in the winning set is the same for all candidates. Since $p_i$ is the same for all candidates and associated utilities, let $p_i = p$ so that we have

$$\mathbf{E}(\mathbf{U}_a) = \mathbf{p} \sum_{i=1}^{n_a} \mathbf{u}_i$$

In order to simplify the computations for $\mathbf{E}(\mathbf{U}_a)$, we use the average of the above threshold utilities by dividing their sum by $n_a$.

$$\mathbf{E}(\mathbf{U}_a) = \mathbf{p}\left(\frac{1}{\mathbf{n}_a}\right) \sum_{i=1}^{n_a} \mathbf{u}_i$$

Therefore, $\mathbf{E}(\mathbf{U}_a) = \mathbf{p}(\mathbf{u}_a/\mathbf{n}_a)$

We define p as the probability that at least one of the individual voter's above threshold candidates is in the winning set. This is completely appropriate because in a multi-member political district it is desirable that at least one member, hopefully more, is in the winning set to represent a particular voter. In the economic sense, Arrow's definition was that only one “commodity bundle” would be available to the consumer out of a number of possibilities in the winning set.

This problem can be modeled as a ball and urn problem containing white and black balls. The candidates above threshold are identified with white balls and the candidates below threshold are identified with black balls. We posit a “picker” that picks balls one at a time out of the urn and places the white balls in the winning set.

The mathematics for this can be expressed by the hypergeometric function:

$$\frac{\binom{\mathbf{K}}{\mathbf{k}}\binom{\mathbf{N}-\mathbf{K}}{\mathbf{y}-\mathbf{k}}}{\binom{\mathbf{N}}{\mathbf{y}}}$$

In probability and statistics, the hypergeometric distribution is a discrete probability distribution that describes the probability of k successes in y draws, without replacement, from a finite population of size N containing exactly K successes, wherein each draw is either a success or a failure. A success is identified as picking one of the candidates above threshold (a white ball) to be in the winning set.

In terms of the ball and urn problem we identify N with the total number of balls in the urn, K with the number of white balls and k with the number of white balls picked out of y draws.

Therefore, in terms of our terminology, y = m, the number of candidates selected, N = n, the total number of candidates, K= number of candidates above threshold and k = number of candidates above threshold selected for the winning set in m picks. Let p = probability that at least one ball chosen is white (above threshold) which equals one minus the probability that at each stage a black ball is chosen.

**p(k above threshold candidates selected out of m picks) =**

$$\frac{\binom{\mathbf{n_a}}{\mathbf{k}}\binom{\mathbf{n}-\mathbf{n_a}}{\mathbf{m}-\mathbf{k}}}{\binom{\mathbf{n}}{\mathbf{m}}}$$

We want to determine where to place the threshold so as to maximize the average expected utility of those candidates above threshold. The probability we are concerned with is the probability that at least one of the candidates selected to be in the winning set is above threshold. The expected utility then is equal to one minus the probability that every candidate selected is below threshold times the average above threshold utility, $u_a/ n_a$..

To simplify the discussion, let us assume initially that the values of the possible utilities are uniformly spread from –1 to +1 in accordance with the spacing,
2/(n-1).

Therefore,

$$\mathbf{E(U_a) = p(u_a/n_a)}$$

For m > 1, we want to place the threshold in such a way as to maximize the average expected value of utility for those candidates above threshold for the individual voter/worker-consumer under consideration. We do the computations for every possible threshold to determine which threshold is best [i.e.] which threshold results in the maximum value of average expected utility. All utilities and associated candidates above threshold will be increased to +1, and those below threshold will be decreased to –1. The results for all candidates will then be tallied over all voters. **Maximizing individual voter satisfaction or utility has to do with the correct placement of the threshold.**

**p (at least one above threshold candidate is selected) =**

**1 – p (every candidate selected is below threshold) =**

$$1-\frac{\binom{n_a}{0}\binom{n-n_a}{m-0}}{\binom{n}{m}}$$

$$=1-\frac{\binom{n-n_a}{m}}{\binom{n}{m}}$$

$$=1-\frac{(n-m)(n-m-1)\dots(n-n_a-m+1)}{n(n-1)\dots(n-n_a+1)}$$

In general we have

$$p = 1 - [1-(n_a/n)][1-n_a/(n-1)]\dots[1-n_a/(n-i)]\dots[1-n_a/(n-m+1)]$$

Let's do an example with m = 1 which should check with the previous result from Smith and Lawrence for range/approval hybrid voting.

$$p=1-\frac{n-n_a}{n}$$

$$=\frac{n_a}{n}$$

Expected value of utility = $\mathbf{E(U_a) = p(u_a/n_a) = (n_a/n)(u_a/n_a) = u_a/n}$

If we place the threshold just under –1, $n_a = n$, $p = 1$, $u_a = 0$, $E(U_a) = 0$.

If we place the threshold just under +1, then

$n_a = 1$, $u_a = 1$, $p = 1/n$ and $E(U_a) = 1/n$.

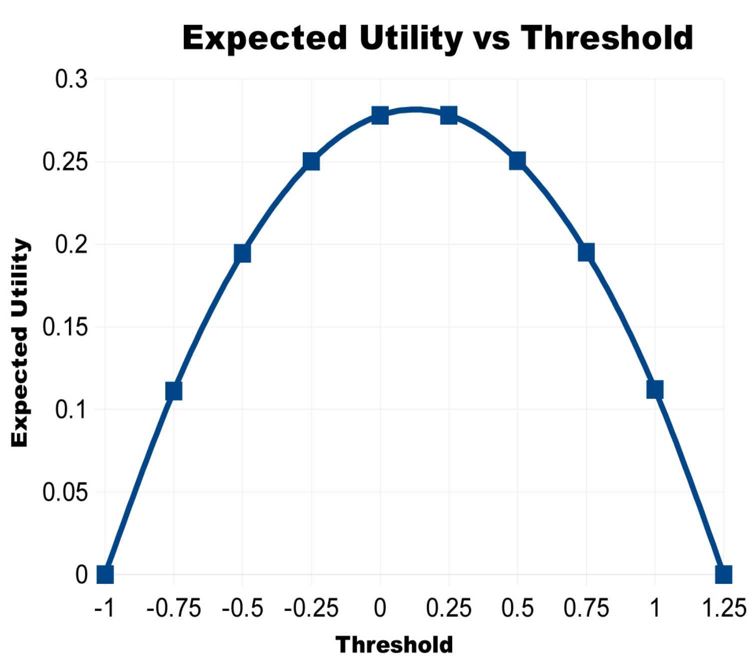
When the threshold is just over +1, $n_a = 0$, $u_a = 0$, $p=0$ and $E(U_a) = 0$.

Let's do an example for the following data set:

$u_i = \{-1,-3/4,-1/2,-1/4,0,1/4,1/2,3/4,1)$

| | | | |
|---|---|---|---|
| For threshold under -1: | p= 1, | $u_a/n_a = 0$, | $E(U_a) = 0$ |
| For threshold under -3/4: | p= 8/9, | $u_a/n_a = 1/8$, | $E(U_a) = 1/9$ |
| For threshold under -1/2: | p= 7/9, | $u_a/n_a = (7/4)(1/7) = 1/4$, | $E(U_a) = 7/36$ |
| For threshold under -1/4: | p= 6/9, | $u_a/n_a = (9/4)(1/6) = 9/24$, | $E(U_a) = 1/4$ |
| For threshold under 0: | p= 5/9, | $u_a/n_a = (10/4)(1/5)= 10/20$, | $E(U_a) = 10/36$ |
| For threshold under 1/4: | p= 4/9, | $u_a/n_a = (10/4)(1/4)= 10/16$, | $E(U_a) = 10/36$ |
| For threshold under 1/2: | p= 3/9, | $u_a/n_a = (9/4)(1/3) = 9/12$, | $E(U_a) = 1/4$ |
| For threshold under 3/4: | p= 2/9, | $u_a/n_a = (7/4)(1/2) = 7/8$, | $E(U_a) = 7/36$ |
| For threshold under 1: | p= 1/9, | $u_a/n_a = 1$, | $E(U_a) = 1/9$ |
| For threshold over 1: | p=0, | $u_a/n_a = 0$, | $E(U_a) = 0$ |

Therefore, expected utility is a maximum when the threshold is just under $u_i = 0$ , $n_a = n/2$. This agrees with the former analysis by Smith and Lawrence since the threshold is placed just under the mean. The graph is as follows:



**Now consider m =2.**

According to the formula,

$$p = \frac{\binom{n-n_a}{2}}{\binom{n}{2}}$$

$$= \frac{(n - n_a)(n - n_a - 1)}{n(n - 1)}$$

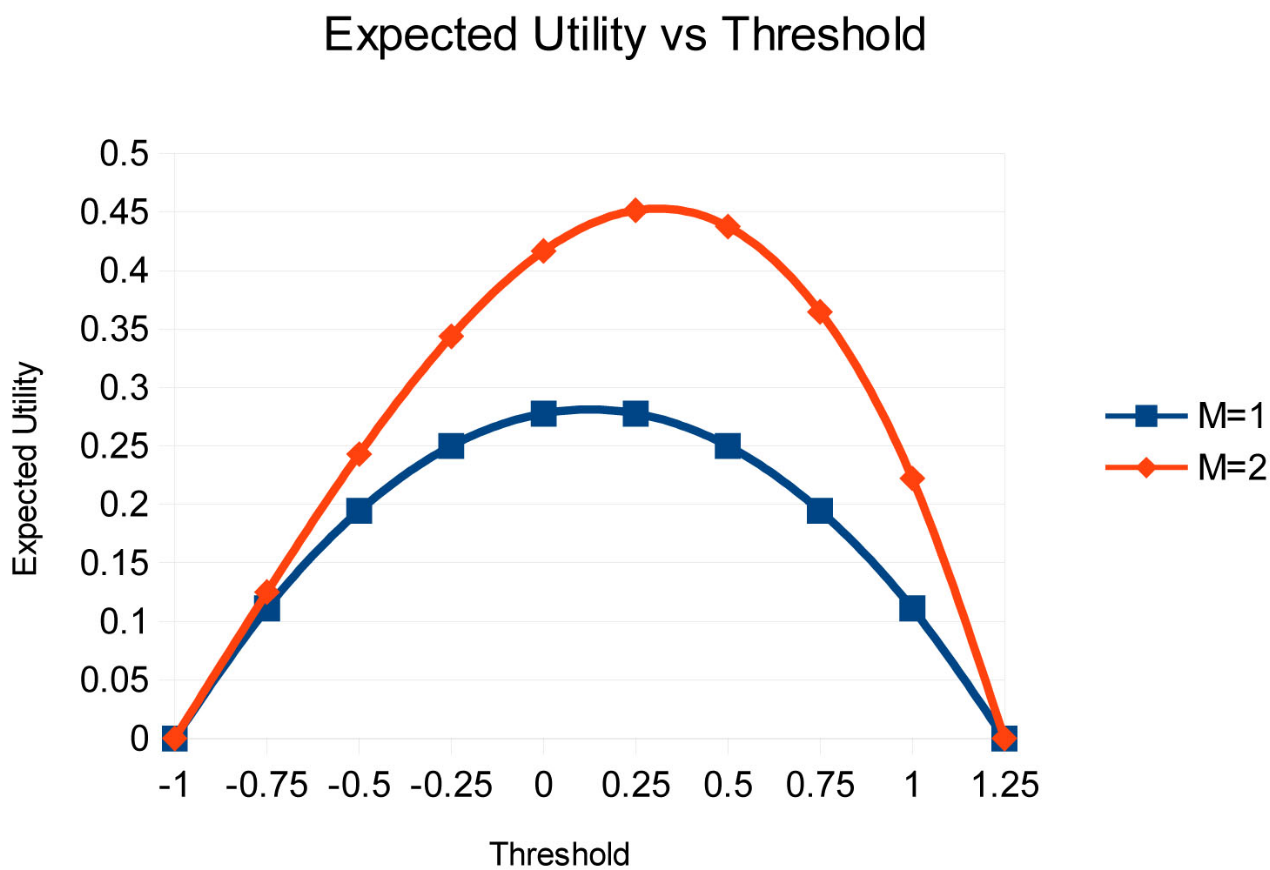
For m = 2, we have p = 1 – (Probability of a black ball drawn on first try)(Probability of a black ball being drawn on second try) = 1 – [(1 /n)(n – $n_a$ )][(1/(n–1)(n – 1 - $n_a$)]

**$p = 1 - [(n- n_a)/n][(n - 1 - n_a)/(n-1)] = 1 - [1-(n_a/n)][1- n_a/(n-1)]$**

For the set $u_i$={-1,-3/4,-1/2,-1/4,0,1/4,1/2,3/4,1), we have

| | | | |
|---|---|---|---|
| For threshold under -1: | p= 1, | $u_a/n_a$ = 0, | $E(U_a)$ = 0 |
| For threshold under -3/4: | p= 1, | $u_a/n_a$ = 1/8, | $E(U_a)$ = .125 |
| For threshold under -1/2: | p= 70/72, | $u_a/n_a$ = (7/4)(1/7) = 1/4, | $E(U_a)$ = .243 |
| For threshold under -1/4: | p= 66/72, | $u_a/n_a$ = (9/4)(1/6) = 9/24, | $E(U_a)$ = .344 |
| For threshold under 0: | p= 60/72, | $u_a/n_a$ = (10/4)(1/5)= 10/20, | $E(U_a)$ = .417 |
| For threshold under 1/4: | p= 52/72, | $u_a/n_a$ = (10/4)(1/4)= 10/16, | $E(U_a)$ = .451 |
| For threshold under 1/2: | p= 42/72, | $u_a/n_a$ = (9/4)(1/3) = 9/12, | $E(U_a)$= .438 |
| For threshold under 3/4: | p= 30/72, | $u_a/n_a$ = (7/4)(1/2) = 7/8, | $E(U_a)$ = .365 |
| For threshold under 1: | p= 16/72, | $u_a/n_a$ = 1, | $E(U_a)$ = .222 |
| For threshold over 1: | p=0, | $u_a/n_a$ = 0, | $E(U_a)$ = 0 |

Here is the graph for m=1 and 2:




We can see that the peak has shifted to the right and upwards indicating that the threshold has shifted up towards greater utilities and the expected utility is greater.

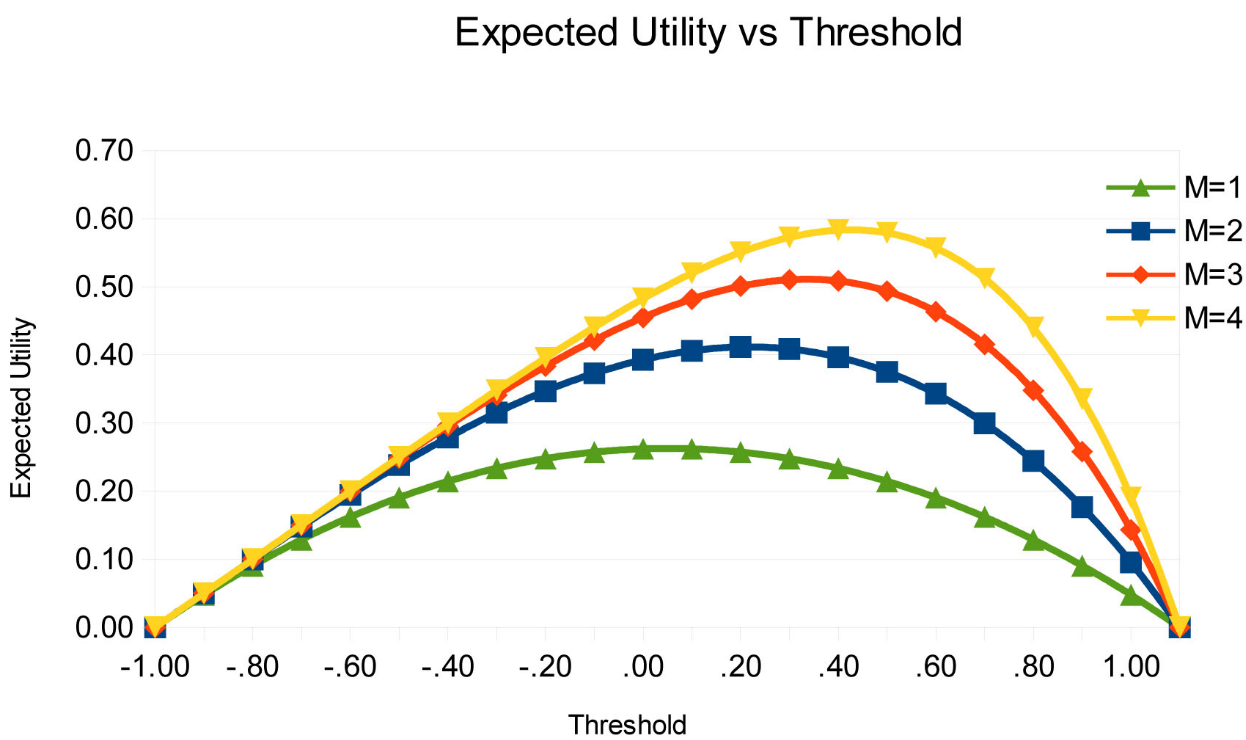
As m increases, the individual voter under consideration has an increased chance of winning in the sense that one of their above threshold candidates becomes one of the **possible realized outcomes.** Their expected utility or satisfaction also increases.

## M = 1, 2 , 3 ,4

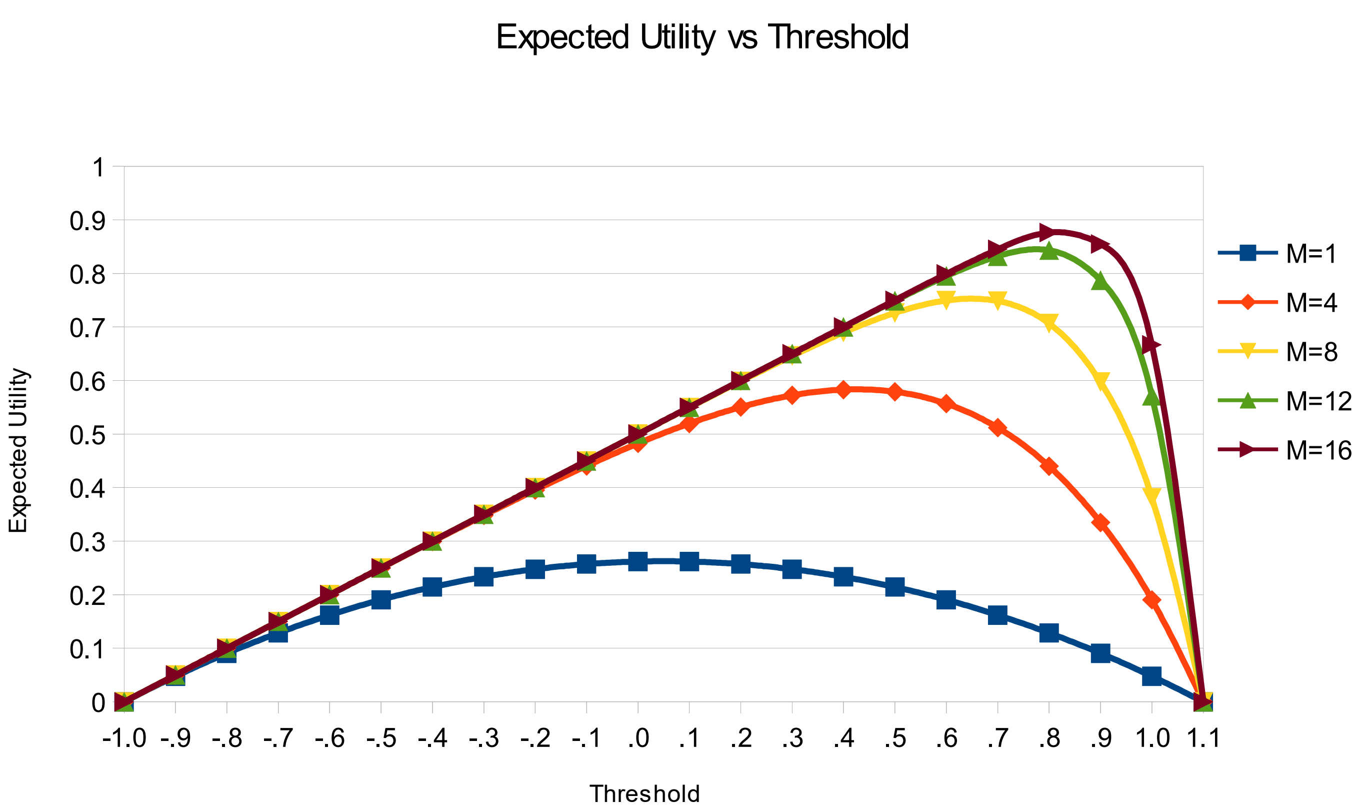
Now we increase the data set as follows

$u_i$ = {-1, -.95, -.9, …, -.05, 0, .05, …, .9, .95, +1}

The graph is the following:




**The following graph shows the thresholds for higher values of m:**



The individual voter/worker-consumer under consideration would adjust the threshold based on the value of m, the number of possible realized alternatives, choosing it to be just under the value of $u_i$ for which average expected utility is a maximum. The individual would advance all candidates with a utility greater than that value to +1. All candidates with a lesser value of utility would have their utilities decreased to –1.

Again the winners are chosen by summing the votes for each individual candidate across all voters. When there are m possible realized outcomes, the top m vote getters are declared winners, and an individual voter is assumed to have picked a winner if at least one of their above threshold candidates is one of the realized outcomes. For example, if there are m possible realized outcomes of baskets of consumer items, an individual will choose that basket which has the highest utility for them out of the m winners. If none of the m winners was an above threshold choice for that individual, they would just choose that basket among the m realized outcomes with highest utility for them. Although the analysis proceeds considering one individual voter, the social decision function can compute thresholds for all voters and apply the optimal strategy for each before summation of votes.

Each utility represents a vote for a particular candidate expressed as a real number between –1 and +1. Strategically increasing a particular candidate's utility increases the probability that that candidate will be elected, and strategically decreasing a candidate's utility decreases the probability that that candidate will be elected.

The problem easily generalizes to the case in which the utilities associated with the candidates are not evenly distributed over the range of utilities since only the average value of utility above threshold is used in the calculations. The values of probability used in the calculations depend on the number of candidates above threshold. These numbers can change with the individual but can easily be incorporated into the calculations.

**<u>Proofs</u>**

Theorem 1: For any given threshold, if we increase the number of alternatives, m, the expected value of utility increases.

$$p_a = 1 - [1- (n_a/n)][1- n_a/(n-1)]...[1- n_a/(n-i)]...[1- n_a/(n-m+1)]$$

$$1 \leq n_a \leq n, i < m$$

For m=1, $p_a = [1- (n_a/n)] = [n - n_a]/n$

for m > 1, $p_a$ equals 1 minus a product of fractions. The more m is increased, the more factors there are in this product and the closer $p_a$ becomes to 1.

Therefore, $p_a$ (for m > 1) > $p_a$ (for m = 1)

For a given threshold, $u_a$ remains constant as m increases.

Therefore, $E(U_a)= p_a u_a$ (for m > 1) > $p_a u_a$ (for m = 1)

The proof easily extends to prove that the expected value of utility for m > 1 is greater than the expected utility for m-1.

Theorem 2: For a given threshold, $E(U_a)$ can be maximized by increasing m and is equal to the value of $u_a$ at that threshold.

As the threshold increases for a given m, $u_a$ increases.

Let k = threshold index = $n - n_a$, $0 < k < n – 1$, $-1 < u < +1$

$\mathbf{E_{ak} = p_{ak} u_{ak}}$

$\mathbf{p_{ak} = 1 - [k /n)][(k-1)/(n-1)]...[(k-i)/(n-i)]...[(k-m+1)/(n-m+1)]}$

The kth term of the series $u_{a0}, u_{a1}, u_{a2}, \ldots, u_{a(n-1)}$ is

$$\mathbf{u_{ak} = \frac{\sum_{k}^{n-1} u}{n-k}}$$

$$u_{ak} = \frac{[n-1-2(k-1)] + [n-1-2(k-2)] + \ldots + [n-1-2(k-i)] + \ldots + n-1}{(n-1)(n-k)}$$

**for $0 < k < 9$, $0 < i < 9$, $k - i \geq 0$,**

$$u_{ak} = \frac{k(n-1)}{(n-1)(n-k)} - \frac{2\sum_{i=1}^{k-1}(k-i)}{(n-1)(n-k)}$$

$$\sum_{i=1}^{k-1}(k-i) = \sum_{i=1}^{k-1}k - \sum_{i=1}^{k-1}i$$

$$\sum_{i=1}^{k-1}k = k(k-1)$$

$$\sum_{i=1}^{k-1}i = \frac{k(k-1)}{2}$$

$$\sum_{i=1}^{k-1}(k-i) = k(k-1) - \frac{k(k-1)}{2} \quad = \frac{k(k-1)}{2}$$

$$u_{ak} = \frac{k(n-1) - k(k-1)}{(n-1)(n-k)}$$

$$u_{ak} = \frac{kn - k - k^2 + k}{(n-1)(n-k)} = \frac{k(n-k)}{(n-1)(n-k)} = \frac{k}{n-1}$$

$$E_{ak} = p_{ak}u_{ak} = [1-(\frac{k}{n})(\frac{k-1}{n-1})\ldots(\frac{k-i}{n-i})\ldots(\frac{k-m+1}{n-m+1})][\frac{k}{n-1}]$$

$k < n$, $k > m-1$, $m < n$

$$p_{ak} = 1-(\frac{k}{n})(\frac{k-1}{n-1})\ldots(\frac{k-i}{n-i})\ldots(\frac{k-m+1}{n-m+1})$$

The second term in $\mathbf{p_{ak}}$ can be driven to zero by increasing m.
If m is sufficiently great, $\mathbf{p_{ak} = 1}$ **for all k and** $\mathbf{E_{ak} = k/(n-1) = u_{ak}}$.

## Conclusions

Warren D Smith and John Lawrence have proved that for the case of one possible realized outcome with a large number of voters, all of whom vote randomly (hence all candidates equally likely to win), your strategically best approval vote is to approve those candidates whose utility exceeds a threshold, T, (raising them to +1) and disapprove the others (lowering them to −1), and your best choice of that threshold T is the *arithmetic mean* utility (for you) of all the candidates. Therefore, range/approval hybrid voting has been proven to be optimal for this case. This paper has shown that range/approval hybrid voting is also optimal for the general case of an arbitrary number of possible realized outcomes after adjusting the threshold. We show how to optimally compute the position of the threshold.

Let m = the number of possible realized outcomes, $p_i$ = the probability of winning for each of i candidates and $u_i$ ($-1 \leq u_i \leq 1$) the utility associated with each candidate. Since no knowledge of the statistics is assumed, $p_i = p$ for all candidates. We assume that what is important to the individual voter is that at least one of their above threshold candidates is chosen to be in the winning set. The threshold is placed in such a way as to maximize the expected value of average utility for those candidates above threshold for each individual – $E(pu_a/n_a)$, where $u_a/n_a$ is the average utility of candidates above threshold. After the threshold has been chosen, utilities that exceed that threshold are strategically maximized to plus one while those below the threshold are strategically minimized to minus one. The values are then summed or tallied for each candidate, and the candidates with the m highest totals are chosen to be in the winning set.

Since the optimal strategy for each individual voter is known, strategy can be taken out of the hands of individuals and placed in the social decision function itself which applies the optimal strategy for each individual before their sincere vote is added to the tally. Any gains from strategizing are thus distributed equally throughout the electorate. This makes it impossible for an individual to gain anything by voting insincerely. As the number of possible realized outcomes increases, the efficacy of strategy decreases since the individual chooser is more likely to get an outcome they prefer regardless of strategy. Thus Arrow's Impossibility Theorem as well as Gibbard and Satterthwaite's concerns about manipulation have been averted. This theory represents a meta-theory from which both political and economic solutions can be derived and unifies the split in social choice theory between political and economic decision making.

We have shown that both political and economic utility or satisfaction increases as the number of possible realized outcomes - either of an election process or of worker/consumer choices – increases. Arrow's Impossibility Theorem gave a theoretically endorsed superiority to winner take all, majority rule single member districts which is the only kind of voting system that isn't manipulable. By the same token there was a tacit endorsement of the capitalist economic system since according to the Theorem, there was no rational method of choosing or assigning economic outcomes. This paper challenges those assumptions and asserts that there is a rational and nonmanipulable basis for aggregating individual choices into rational social decisions. Thus Arrow's Impossibility Theorem has been transcended.

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