Utilitarian Approval Social Choice

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This is a PrePrint

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February 25, 2023

Abstract

Utilitarian voting (UV) and approval voting (AV) are well known voting/choosing systems. Any utilitarian system that amalgamates votes or choices has hitherto been thought to be impossible or one susceptible to strategy. Utilitarian approval voting (UAV) involves placing a threshold in the individual utilitarian rating profiles and converting all those above threshold into approval style votes. The system presented here is designed to disincentivize individual choosers from choosing insincerely. The system itself maximizes the efficacy of each individual input. The information is processed in such a way as to alleviate concerns about interpersonal comparisons of utility. A theorem is proved that this mechanism satisfies Arrow's five rational and normative conditions, and, because of the more finely tuned data, it is even more robust normatively. UAV also produces the utilitarian winner(s), the one(s) which maximizes social utility, and a Rawlsian maximin condition can be applied such that those whose outcome utilities are the lowest can be raised to at least a minimum level.

Introduction

In Social Choice and Individual Values, Kenneth Arrow (1951: p. 1) showed that there is no way to make social decisions based on the amalgamation of individual ones, assuming certain rational and normative conditions are met. Arrow's result, formerly called the *paradox of voting,* was first discovered by the Marquis de Condorcet (1785). Condorcet's paradox showed that majority preferences can become intransitive when there are three or more alternatives. Arrow mathematized Condorcet's insight.

Gibbard and Satterthwaite concurred with Arrow and proved that any social choice system that was strategy proof was also impossible. Gibbard (1973: p. 587) states: “An individual 'manipulates' the choosing scheme if, by misrepresenting his preferences, he secures an outcome he prefers to the 'honest' outcome - the choice the community would make if he expressed his true preferences." Satterthwaite (1975: p.188) showed that the requirement for voting procedures of strategyproofness and Arrow’s requirements for social welfare functions are equivalent: "a one-to-one correspondence exists between every strategy-proof voting procedure and every social welfare function satisfying Arrow’s four requirements."

Jackson (2001: p. 2) states: "Often, one thinks of the desired outcomes as the given and analyzes whether there exist game forms for which the strategic properties induce individuals to (always) choose actions that lead to the desired outcomes." This paper discusses a game form for which the strategic properties induce individuals to choose actions that lead to the desired outcome – a possible social choice – while disincentivizing them from choosing strategically as individuals. We show that this mechanism also satisfies Arrow's rational and normative conditions.

Gibbard's results were based only on the possibility that someone could use strategy if they were astute enough to stumble on a way to do so. (1973: p. 590) "Note that to call a voting scheme manipulable is not to say that, given the actual circumstances, someone is really in a position to manipulate it." Only the possibility exists in an elaborate mathematical structure. Gibbard doesn't assume that there is any formularizable or identifiable strategy that an individual chooser could use to manipulate the system. Other writers have pointed out this difficulty: (Meir et. al.: p. 149) "In other words, computational complexity may be an obstacle that prevents strategic behavior." By contrast, we analyze a situation in which an actual identifiable strategy exists which can be known both to the individual chooser and to the mechanism, which amalgamates or processes the choices, itself. If the mechanism does the strategizing for each individual, there is no incentive for the individuals to do so.

Arrow's, Gibbard's and Satterthwaite's analyses are deterministic while the problem of manipulability is inherently probabilistic. In an actual election it would be impossible for a voter to know the ideal strategy unless they knew how every other voter was going to vote exactly. Polling, however, can provide some information of a probabilistic nature about other voters. We incorporate the fundamentally probabilistic nature of the choosing process in our analysis, and the mechanism we develop is generalizable to the situation in which polling data is available. Other writers (Cranor,1996; LeGrand, 2008) have also sought to develop systems such as Declared-Strategy Voting which attempts to "elicit more sincere preferences from voters ... to find a winning alternative in such a way that voters would be unlikely to gain a superior result by submitting insincere preferences."

Aki Lehtinen (2011: p.376) concludes that Arrow's Impossibility Theorem is not relevant in the final analysis:

> “Arrow’s impossibility result and the closely related theorems given by Gibbard and Satterthwaite are unassailable as deductive proofs. However, we should not be concerned about these results because their most crucial conditions are not justifiable. Fortunately, we know that strategy-proofness is usually violated under all voting rules and that IIA [Independence of Irrelevant Alternatives] does not preclude strategic voting.”

Unlike Lehtinen, we do not dispute the Arrow and Gibbard-Satterthaite analyses and conclusions. Their mathematics is impeccable. Instead, by thinking outside the box, we analyze a social choice mechanism which accomplishes what Arrow, Gibbard and Satterthwaite purportedly set out to accomplish – a system that produces a social choice based on individual inputs and which exemplifies certain rational and normative criteria including strategyproofness. The mechanism analyzed here accomplishes this in a manner that not only is more realistically implementable in terms of actual voting/choosing systems but is also more robust normatively.

A major stumbling block for the development of utilitarian social choice systems regards the issue of interpersonal comparisons. It has been thought that scales which measure the utilities of individuals are incompatible, and that any scale chosen, upon which all individuals are supposed to rate their utilities, would be arbitrary. Arrow (1951: p. 9) states: "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility." Thus, according to Arrow, any individual input must be based on individual preference rankings of the form *aRbRc*..., meaning *a* is preferred or indifferent to *b*, *b* is preferred or indifferent to *c* etc. Although "comparisons in the measurability of individual utilities" may have no meaning when done by an outside observer, the assertion of utilities by individuals themselves on a scale of their own choosing certainly does. Furthermore, basing all inputs on the form *aRbRc* tacitly assumes that there is an equality of utility scales among all inputs.

We assume that choosers can place their respective utilities for alternatives on a scale of their own choosing on a line consisting of the set of all non-negative real numbers, $\mathbb{R}_{\geq 0}$, and also choose the end points. In general there will be a utility for each possible alternative specified by each chooser. We will show that, for the mechanism modeled here, any affine linear transformation of an individual's set of utility ratings will yield the same social choice result, and, therefore, it doesn't matter which scale an individual chooses. This is not to say that the utility scale chosen by an individual is not meaningful to the individual themselves, but just that, whatever scale they choose, their contribution to the final output of the mechanism we analyze will be the same.

We develop a social choice mechanism that is utility based, but which overcomes the objections of arbitrariness of individual utility scales, is strategyproof and also meets an upgraded version of Arrow's normative and rational criteria.

Utilitarian and Approval Choosing

Utilitarian and approval choosing are exactly analogous to utilitarian voting (UV) and approval voting (AV) (Brams and Fishburn, 1983), and, therefore, “voting” and “choosing” are used interchangeably for the purposes of this paper. Also the words “alternative” and “candidate” will be used interchangeably. In Social Choice and Individual Values, Arrow (1951) clearly intends to incorporate both political and economic decision making in his analysis. Political decision making can be characterized by a social decision that applies to everyone while economic decision making can be characterized by a social decision that is comprised of individual outcomes for everyone.

Arrow sets up the problem so that each individual chooser orders or ranks all alternatives and then society is required to come up with an ordering that is best according to his stated criteria. He states (Arrow, 1951: p. 11-12) “In the theory of consumer's choice each alternative would be a commodity bundle; ... in welfare economics, each alternative would be a distribution of commodities and labor requirements. … in the theory of elections, the alternatives are candidates.” In today's economy rather than alternatives being commodity bundles, they might instead be cash payments and labor requirements.

Claude Hillinger (2005: pp. 295-321) has made the case for utilitarian voting:

> “There is, however, another branch of collective choice theory, namely utilitarian collective choice, that, instead of fiddling with Arrow’s axioms, challenges the very framework within which those axioms are expressed. Arrow’s framework is *ordinal* in the sense that it assumes that only the information provided by individual orderings over the alternatives are relevant for the determination of a social ordering. Utilitarian collective choice assumes that individual preferences are given as *cardinal* numbers; social preference is defined as the sum of these numbers.”

The difference between Hillinger's statement and the mechanism considered here is that social preference is *not* defined as the sum of cardinal numbers. There is a unique transformation done by the mechanism itself *for each voter* from their cardinal inputs to their AV style contribution to the social choice output. Hence, the system we examine is a utilitarian approval hybrid (UAV).

Lehtinen (2015: p.35) has shown that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". The utilitarian winner is the one that maximizes the social utility of the social choice. Therefore, the mechanism described in this paper should accomplish two things: sincere voting behavior on the part of individuals *and* increased selection of the utilitarian winner or winners compared to other voting systems. While Lehtinen abandons the Arrow and Gibbard-Sattherwaite conditions in the interests of increased social utility, strategyproofness is not violated if the mechanism, which amalgamates the individual voting/choosing information, itself applies the strategy instead of the individual choosers. It is shown elsewhere (Lawrence, 2024: pp. 9-11) that in fact the Optimal Choice Mechanism (OCM) considered here does in fact result in the utilitarian winner(s). Therefore, it is possible to specify a maximin condition which raises the utilities of the least well off while lowering somewhat the utilities of the utilitarian winner(s). Therefore, instead of making adjustments in the choosing process at the beginning, the adjustments are made at the output of the procedure which makes a lot more sense.

Lehtinen (2015: p. 39) also argues that interpersonal comparisons "can be made in a methodologically acceptable way in evaluating the performance of voting rules if the same comparison is made under every voting rule." The issue is that a methodologically acceptable way of combining individual inputs is made such that the social choice results are the same regardless of the scale of the inputs. The issue of interpersonal comparisons is demonstrably moot for the implementation of the social choice mechanism considered here because an affine linear transformation of each individual's utilitarian style input does not affect their contribution to the social choice results.

The method constructed in this paper and inputs information from the individual choosers in the form of preference ratings over each alternative and outputs information in the form of complete social preference rankings of the alternatives from which social and individual ratings can be derived since the underlying individual ratings are known. From these social preference rankings, an unordered winning set, *W,* of size *m*, is constructed consisting of those alternatives with the top *m* rankings. The utility of the winning set for each individual, which is the summation of their utilities over each alternative in the winning set divided by *m*, can be computed since we know from the individual inputs how each individual rated each alternative. Summing utilities over all voter/choosers and dividing by *q*, the number of voter/choosers, gives the social utility of the winning set. In the case of elections the winning set would be the members of a legislative body or in the case of $m = 1$, a President. In the case of welfare economics, rather than being a distribution of commodities and labor requirements as Arrow suggests, the more likely scenario in an employer/employee situation would be a distribution of cash payments and labor requirements.

In order to overcome the Gibbard-Sattertwaite theorems, which maintain that every choosing system for which an individual chooser can use strategy to improve the outcome for themselves violates Arrow's conditions, we choose a social choice mechanism which itself implements the optimum strategy for each individual assuming that that strategy consists of each individual's choosing in such a way as to maximize the expected utility of their contribution to the winning set. We assume a completely random distribution of chooser utility profiles and that each chooser has no knowledge of the utility profiles of other choosers. The optimum strategy can be known both to the individual chooser and to the mechanism which amalgamates the choices itself. If the mechanism applies the optimum strategy to each chooser's input, then the individual chooser is disincentivized from doing so and *is* incentivized to submit their sincere utilities.

Each chooser rates each candidate by assigning to him or her a real number between zero and one. The utility profile, U_j, consists of this set of ratings where j signifies the specific individual. The mechanism described here involves the placing of an individualized threshold in the monotonically increasing and unrestricted utility profile which is submitted by each individual chooser. Each candidate above this threshold is given an approval style vote of "+1", and each candidate below threshold is given an approval style vote of "0". This strategy can be seen as the extension of the optimal strategy when there are only two candidates which is to give the one with higher utility an approval style vote of "1" and the one with lower utility an approval style vote of "0." The threshold is placed such that the expected average utility of those candidates above threshold is a maximum. When there are more than two candidates and $m \geq 1$, we vote approval style and give the candidates above threshold a rating of "1" and the rest a rating of "0".

In approval voting (AV) the placing of the threshold is left up to the voter. In this paper we calculate a more exact rationale for placing the threshold taking into account the probability of being elected to the winning set for each candidate, and other factors such as the number of candidates and the size of the winning set in order to maximize the expected value of utility of each individual's contribution to the winning set for themselves. Heuristically, the threshold should be set higher in an individual's utility profile if the ratio of m to n is small and lower if the ratio of m to n is large. Also the utilities that each voter has for the candidates should be mitigated by the probabilities that those candidates will actually be elected. Those probabilities are set by the voters as a whole, and they must be used in the analysis. If the set of candidates above threshold has a low probability of being elected, then perhaps the threshold should be placed somewhere else.

As the threshold increases, there are less candidates above threshold, the average utility rating of the set of candidates above threshold increases, and the probability of selecting randomly any particular candidate in this set decreases. Conversely, as the threshold decreases, the number of candidates above threshold and the probability of random selection of one of them increases while the average utility rating of that set of candidates decreases. The mechanism we explore here chooses the optimum threshold, individualized for each chooser, to be just under that utility such that the expected value of their contribution to the winning set, *W*, is a maximum.

Formal Statement of System Parameters

We first define the following sets:

i) $V = \{v_1, v_2, \ldots v_q\}$ is a set of choosers of size q, where $v_j \in V$ denotes the j^{th} chooser.

ii) $C = \{c_1, c_2, \ldots, c_n\}$ is an ordered set of candidates of size n; candidates appear on the ballot in $c_1, c_2, \ldots, c_n$ order. $c_i \in C$ denotes the i^{th} candidate.

iii) $X = \{x_1, x_2, \ldots x_n\}$ $x_i = \{\mathbb{N}^0\}$, is a set of non-negative integers. X represents the cumulative votes for candidates as they appear on the ballot.

iv) $Y = \{y_1, y_2, \ldots y_n\}$ is the set which orders the candidates by the number of votes received by each candidate. $y_1 R y_2 R \ldots R y_n$.

v) $W = \{w_1, w_2, \ldots, w_m\}$ is a set of candidates of size $m < n$ representing the unordered winning set.

vi) $C_j = \{c_{1j}, c_{2j}, \dots c_{nj}\}$ is the ordered set of preferences for alternatives of the j^{th} voter. $c_{1j} R_j c_{2j} R_j c_{3j} R_j c_{4j}, \dots, c_{n-1j} R_j c_{nj}$

vii) $B_j = \{b_{1j}, b_{2j}, \dots, ,b_{nj}\}$ is a set of approval style votes in order of the j^{th} voter's candidate preferences. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$

viii) $O_j = \{o_{1j}, o_{2j}, \dots\}$ O_j is the set of candidates given approval style votes of "1" by voter j, called the optimal set.

ix) $U_j = \{u_{1j}, u_{2j}, \dots u_{nj}\}$ is a set of utilities of size n, with $u_{1j} \geq u_{2j} \geq \dots \geq u_{nj}$ and $0 \leq u_{ij} \leq 1$, $\forall$ i,j. U_j is the utility profile of the j^{th} voter after applying an affine linear transformation to their submitted set of utilities. u_{ij} is the utility of candidate c_{ij}

x) $T_j = \{t_{1j}, t_{2j}, \dots t_{nj}\}$ is a set of thresholds of size n such that $t_{1j} \geq t_{2j} \geq \dots \geq t_{nj}$ and $0 \leq t_{ij} \leq 1$, $\forall i, j$.

xi) $U_{aj} = \{u_{a1j}, u_{a2j}, \dots, u_{anj}\}$ is the set of sums of utilities above threshold for each chooser, j. u_{aij} is defined as the sum of utilities above threshold t_{ij} for voter j, $\forall i, j$. The sum of utilities above threshold is computed for each of the n thresholds. n_{aij} is the corresponding number of utilities above threshold $\forall i, j$. u_{aij}/n_{aij} is the average utility above threshold.

We now define the following functions:

i) $\tau : C \rightarrow X$ defines an ordered pair, (c_i, x_i) such that $\tau(c_i) = x_i$,the cumulative number of votes for each candidate.

ii) $\alpha : X \rightarrow Y$ α defines an ordered pair (x_r, y_r) such that [$y_r \mathrm{R} y_z$ iff $x_r \geq x_z$]

for $1 \leq r, z \leq n$; *r , z, n integers.*

iii) $\beta : Y \rightarrow W$ such that $\beta(\mathrm{y_i}) = w_i$ for $1 \leq i \leq m$. The function, β, places the top m vote getters in the winning set. If y_m represents a tie with y_{m+z} for $z \geq 1$, ties are resolved randomly so that W is always of size m.

iv) $\chi_j : C \rightarrow C_j$ The function χ_j assigns to each element $c_i \in C$ an element $\chi_j(c_i) = c_{ij}$ such that $c_{1j} \mathrm{R_j} c_{2j} \ldots c_{(n-1)j} \mathrm{R_j} c_{nj}$ for $1 \leq j \leq q$ where $\mathrm{R_j}$ means "is preferred or indifferent to". Each voter, j, orders the set of alternatives according to their preferences.

v) $\eta_j : C_j \rightarrow U_j$ the function η_j assigns to each element $c_{ij} \in C_j$ an element $\eta_j(c_{ij}) = u_{ij}$ where u_{ij} is the utility that is assigned to candidate c_{ij} by voter j.

vi) $\delta_j : C_j \rightarrow B_j$ defines an ordered pair (c_{ij}, b_{ij}) such that $\delta_j(c_{ij}) = b_{ij}$ for $1 \leq j \leq q$ and $1 \leq i \leq n$

vii) $\gamma_j : U_j \rightarrow T_j$ defines the relationship $\gamma_j(u_{ij}) = t_{ij}$ such that $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1, \ \forall \ i, j$

viii) $\phi_{aj} : T_j \rightarrow U_{aj}$ such that $\phi_{aj}(t_{ij}) = u_{aij}$, where $u_{aij} = \sum_{u_{ij} > t_{ij}} u_{ij} \quad \forall \ i, j$

Strategy

We focus now on one particular voter called the focal voter. While Brams and Fishburn (1983: p. 73) "presume that voters' preferences are more or less evenly distributed over the different preference orders for the ... candidates," we determine the best way for a voter with a particular preference order, in this case a utility profile, U_j, to vote. We analyze the focal voter's efficacy in changing the election results using strategy due to their input alone. The strategy involves separating the candidates into two dichotomous sets by placing a threshold in the focal voter's set of utilities such that approval style votes of "+1" are cast for candidates with utilities above threshold and approval style votes of "0" are cast for candidates with utilities below threshold. Once utilities are increased to "1" or decreased to "0", they become prototypical votes, and these are the inputs to the system.

Where this threshold should be depends on the utilities of the candidates themselves and also on the probabilities that the candidates will be in the winning set. We model the focal voter's situation as a ball and urn problem consisting of n black and white balls representing the candidates. We identify the white balls with candidates with utilities above threshold and black balls with candidates with utilities below threshold. Let n_{aij} be the number of candidates above threshold and $n - n_{aij}$ the number of candidates below threshold. We choose randomly m balls out of the urn without replacement and place them in the winning set, W. The probability, p, of k above threshold candidates being in the winning set due to chance alone is given by the hypergeometric function:

$$p(k)=\frac{\binom{n_{aij}}{k}\binom{n-n_{aij}}{m-k}}{\binom{n}{m}}$$

We assume no prior knowledge or polling information regarding candidate probabilities although the analysis can be generalized to the case where polling information is available. Exactly which black or white ball (associated with a particular candidate) is picked is not known. However, the average utility of above threshold candidates, u_{aij}/n_{aij}, can be calculated.

Let u_{Wj} be a random variable which represents the average utility of above threshold candidates in the winning set due to voter j's choice alone so that $0 \leq u_{Wj} \leq 1$, $\forall j$. Then the expected value of average utility of above threshold candidates in the winning set for voter j at threshold t_{ij} is given by

$$E_{t_{ij}}(u_{Wj}) = \sum_{k=1}^{s} \left\{ \left[\frac{\binom{n_{aij}}{k}\binom{n-n_{aij}}{m-k}}{\binom{n}{m}} \right] \left[\frac{u_{aij}}{n_{aij}} \right] \right\}$$

where $s = min\{ m, n_{aij}\}$

We now perform a thought experiment in which we calculate the expected value of average utility for every threshold. So n_{aij} varies from 1 to n. For each value of n_{aij} we randomly withdraw balls from the urn and place them in the winning set. We do this repeatedly to determine the value of expected utility at each threshold, $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1$. Let t_j^* be the optimal threshold which is the threshold which results in the maximization of the expected value of average utility, $Et_{ij}(u_{Wj})$, for voter j. n_j* is the corresponding number of candidates with utilities above that threshold.

Therefore, the opimal threshold is the one for which the following equation holds:

$$E_{t_j^*}(u_{Wj}) = \max\{E_{t_{ij}}(u_{Wj})\}$$

The set of candidates above optimal threshold is called the optimal set, O_j.

$O_j = \delta^{-1}{}_j(B_{ij})$ such that $b_{ij} = 1$. As the threshold is decreased from $t_j^{\star}$, the average utility of the optimal set for voter j decreases because there are more above threshold utilities with lower values of utility under consideration, and the probability of an above threshold candidate being in the winning set increases. As the threshold is increased from $t_j^{\star}$, the probability of an above threshold candidate being in the winning set decreases, and the average utility of the set of candidates above threshold increases. Candidates whose utilities are greater than the optimal threshold, $t_j^{\star}$, will be given the maximum AV vote of "+1", and candidates whose utilities are less than $t_j^{\star}$ will be given the minimum AV vote of "0" $\forall j$. The individual voter's strategy is to give a one vote boost to candidates above threshold which belong to the set for which the voter has the greatest expected average utility.

If the ball and urn experiment were to be performed on each member of the electorate as a whole minus the focal voter, the total number of white balls representing AV votes for each candidate could be computed. With the addition of the focal voter's votes, there is a finite probability that one or more candidates will be elevated to the winning set resulting in a tie or ties with a candidate already in the winning set. The focal voter's AV votes could potentially determine the constitution of the winning set if a candidate's being in the winning set can be determined by a single vote after all other voters have cast their ballots. If ties are resolved randomly, the focal voter could still determine the constitution of the winning set.

When Polling Information Is Known

We associate with each candidate, c_i, a probability of being elected to the winning set, p_i. Again the strategy involves separating the candidates into two dichotomous sets by placing a threshold in the focal voter's set of utilities such that approval style votes of "+1" are cast for utilities above threshold and approval style votes of "0" are cast for utilities below that threshold. We compute the expected value of average utility of those candidates above threshold.

$$E_{>t_{ij}}\left(u_{W_j}\right) = \left(\frac{1}{n_{aij}}\right)\sum_{t_{ij}}^{1} p_i u_{ij}$$

We do this for every threshold t_{ij}. We choose that threshold such that the expected value of average utility is a maximum. The optimal threshold, t_j^*, is the one for which the following equation holds:

$$E_{t_j^*}(u_{wj}) = \max\{E_{t_{ij}}(u_{wj})\}$$

Then all those candidates with utilities above the optimium threshold get approval style votes of "+1", and all those candidates with utilities below threshold get approval style votes of "0".

Counting the Votes

The vote count proceeds by the following algorithm, **σ**:

```
σ: for z = 1, n
     x_z = 0
end z (initializes X)
     for j = 1,q
        for i = 1, n
            b_ij = 0
            if {u_ij ≥ t_j*}  then
               b_ij = 1
               x_i = x_i + 1
        end i
     end j
end σ
```

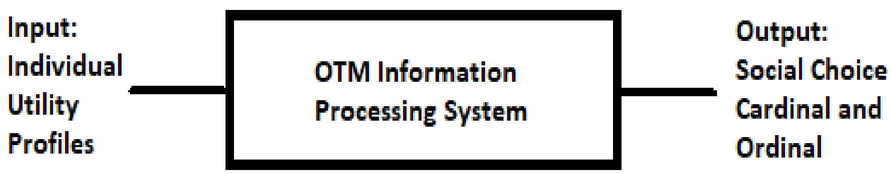
Let $^A u_j$ be the utility of the winning set, *W*, for chooser *j* post-election, and $^A u$ be the social utility of the winning set for all choosers - the utility of the social choice. It is also possible to compute individual and social utilities based on the voters' original utility profiles before the affine linear transformation to $0 \leq u_{ij} \leq 1$.

$$ {}^{A}u_j = \sum_{i=1}^{m} \eta_j \chi_j \tau_i^{-1} \alpha_i^{-1} \beta_i^{-1} (w_i) \qquad {}^{A}\mathbf{u} = \sum_{j=1}^{q} {}^{A}\mathbf{u}_j $$

Optimal Threshold Mechanism

The Optimal Threshold Mechanism (OTM) Information Processing System can be modeled as follows:

Figure 1



The OTM mechanism uses the above analysis to optimize each individual's choice so that they are disincentivized from choosing insincerely. It overcomes Gibbard-Satterthwaite's concerns about strategic choosing by individuals while meeting Arrow's rational and normative conditions as proven in Appendix A. It even upgrades Arrow's normative conditions since more finely tuned cardinal input information is used while Arrow's analysis only involved less precise ordinal information. Moreover, the welfare or utilitarian results for each individual and for society as a whole are measurable. The key is that individuals are disincentivized from voting insincerely because the OTM system strategizes for them. The optimal strategy maximizes the expected value of utility of each individual's contribution to the winning set, *W*. The assumption of utility maximizing is made by other writers (Lehtinen, 2008: pp. 688-704): "Under strategic behaviour voters are assumed to maximise expected utility ... ". The voter's input is the ordered set of candidates C_j and the associated ordered set of utilities U_j. The output is the set Y consisting of the ordered set of all candidates by vote totals from which is derived the winning set, W, which is unordered and consists of $m < n$ candidates. It is assumed that each individual voter specifies an unrestricted, utilitarian style input profile, U_j, which represents their sincere utility ratings for candidates in the set, C.

Optimal Threshold Social Choice is Strategyproof

Since the data is processed in an optimal manner for each individual chooser by the system itself, giving each chooser the optimal strategy, the choosers have no incentive to misrepresent their preferences or to choose insincerely. They would either choose sincerely or the OTM system might process their input in such a way as to give them a suboptimal result. There is no advantage for individuals to misrepresent their preference ratings. The strategy has been placed in the processing of the choices rather than in each individual chooser's hands.

The optimum strategy for each individual is to vote in such a way as to maximize their expected utility, ${}^{A}u_j$, for the winning set. This is done by the OTM system itself by setting an optimal threshold in each individual's sincere utility style input so that each candidate above threshold receives the maximum “vote” and every candidate below threshold receives the minimum “vote”. This maximizes the expected value of utility of the social choice for each individual based on that individual's choice alone. This also effectively turns the utilitarian style inputs into approval style outputs, but the connection with the underlying utilitarian basis of the system is maintained since the original utilities are known to the OTM mechanism and can be used to compute the utility of the social choice for each individual and for society as a whole.

The Issue of Interpersonal Comparisons is Moot

Arrow (1951: p. 10) dwells on the fact that individual utility scales are not compatible. He compares them with the measurement of temperature which is based on arbitrary units and the arbitrary terminal points of freezing and boiling for the Celsius scale and completely different end points for the Fahrenheit scale.

> "Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, their utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choice compatible."

Arrow's analysis is based on how an outside observer would select an indicator to represent each individual. Our analysis is based on how each individual, theirself, would choose their own indicator. As Arrow suggests, we take into account that each individual has a unique utility function. There is no need to (Arrow: p. 12) “choose one out of the infinite family of indicators to represent the individual.” Each individual gets to choose their own indicator. Let's say that, in general, utility can be measured using the set of non-negative points u_{ij} on the real line, $R_{\geq 0}$. It's up to the individual chooser where to place the points, including the end points, corresponding to the utilities of each alternative in the alternative set consisting of n alternatives, $C = \{c_1, c_2, \ldots, c_n\}$. Let's call the end points of some individual's utility scale u_{max} and u_{min}. This will define the scale. There needs not be an actual utility assigned to either of these end points. Since the OTM system optimizes the utility of the social choice for each individual, there would be an optimal threshold above which all utilities are changed to the maximum value and below which all utilities are converted to the minimum value.

For the OTM system in particular, the results will be the same no matter which utility scale each individual chooses since the optimal threshold is a function of n^*_j. Any affine linear transformation of a chooser's utility scale will yield the same results since n^*_j will be the same before and after the transformation. Let an individual express their utilities on a scale of their choice on the real line. For the sake of the analysis, the OTM system preprocesses and converts each individual's input utility scale to one with end points "0" and "1" by means of an affine linear transformation. $f(u) = au + b$ with a,b real numbers.

Sen's (1970) Cardinal Non-Comparability condition(CNC) states that for all $U_j, U'_j \in \mathbb{R}$, one has $f(U_j) = f(U'_j)$ whenever for all $j \in V$, there are real constants κ_j and ν_j, with each $\nu_j > 0$, such that $U'_j \equiv \kappa_j + \nu_j U_j$. The social choice mechanism detailed in this paper is invariant to affine rescaling of utilities since the optimal threshold is a function of n^*_j $\forall$j. This makes it possible to rescale every utility function so that it consists of numbers such that $0 \leq u_j \leq 1$ $\forall$ j. In effect this equalizes all "utils." A utility of "1" from a particular voter/chooser does not have the same meaning as a utility of "1" from some other voter/chooser. This is not any different from assuming inputs of the form R_1, ..., R_n, which Arrow assumes, in the sense that all "utils" have been equalized and utility information has been lost. However, we have assumed that utility information has been uploaded to the OTM system in the form of utilities from each voter/chooser. This makes it possible, after the social choice has been determined, to implement a maximin or leximin condition so that the utilities of those who have the lowest utilities are increased to at least a minimum level. Since we prove later that the OTM mechanism results in the utilitarian winner(s), implementing a maximin or leximin condition diminishes the utility of the social choice in order to insure that each participant has at least a minimum utility at the outcome. This compensates for the fact that as Arrow writes "The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, there is no meaning relevant to wefare comparisons in the measurability of individual utility."

Consequently, Arrow's statement that "the values of the aggregate are dependent on how the choice is made for each individual” is not true. The choice is not *made* for each individual; each individual makes their *own* choice. However, since any scale chosen by each individual will yield the same results, without loss of generality, we can standardize the choosing process by transforming individual scales to the real line between "0" and "+1", preprocessing the data before input to the OTM system.

Amartya Sen (2002: p. 71) stated “... economists came to be persuaded by arguments presented by Lionel Robbins and others (deeply influenced by "logical positivist" philosophy) that interpersonal comparisons of utility had no scientific basis. 'Every mind is inscrutable to every other mind and no common denominator of feelings is possible.' Thus, the epistemic foundations of utilitarian welfare economics were seen as incurably defective." The OTM system demonstrates that there is a sound epistemic basis for a utility based social choice mechanism. Therefore, it is in fact logical positivist *because* it has a sound scientific basis. Arrow and Gibbard-Satterthwaite did not incorporate probability into their analysis. That made their analysis unrealistic in terms of voting and choosing systems and led to their impossibility results.

The OTM Mechanism Chooses the Utilitarian Winner(s)

In fact the Optimal Choice Mechanism (OCM) considered here chooses utilitarian winner(s). Following is a proof by contradiction.

Consider the candidate, $y_m = \beta^{-1}(w_m)$, in the winning set who has the least number of votes, let's say x_m votes. Discounting ties, the next highest ranked candidate, y_{m+1} has at most x_m -1 votes. Replace y_m in the winning set with y_{m+1}. Call this set *W′*. Assume that the set *W′* has greater total utility than the set *W*. Therefore,

$$^{A}\mathbf{u}_j = \sum_{i=1}^{m} \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_{m+1} > \sum_{I=1}^{m} \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_m$$

and

$$^{A}\mathbf{u}_j = \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_{m+1} > \eta_j \chi_j \tau^{-1} \alpha^{-1} \mathbf{y}_m$$

But by assumption, $x_{m+1} = \alpha^{-1}(y_{m+1}) < x_m = \alpha^{-1}(y_m)$

Therefore, the utility of the set, *W′*, is less than the utility of the set, *W*, and the winning set, *W*, represents the set with the highest total utility, the utilitarian winner. QED.

A Maximin Condition is Possible

After the optimal set has been established and the output utilities computed for each individual, the worst off person or set of persons in terms of utility might have their results improved at the expense of a diminution of total social utility. In the debate among principles of distributive ethics, two of the main contenders are contractualism as exemplified by John Rawls (2001), and utilitarianism as represented by John Harsanyi (1977), Amarya Sen (2002), and others. T.M. Scanlon (1982) writes, "Contractualism has been proposed as the alternative to utilitarianism before, notably by John Rawls in *A Theory of Justice*."

In terms of the OTM system, one way to do this is as follows. Starting with the worst off individual or set of individuals, make all possible changes to the winning set and calculate the worst off set's utility and also the total utility after each change has been made. If the worst off set's utility can be increased in such a way that the total utility is not decreased more than that increase, then this would be a possible maximin solution. This is considered in another paper, *Utilitarian Social Choice With a Minimax Provision* (Lawrence, 2024).

Preference Rankings Can Be Converted to Ratings and Vice Versa

Arrow's assumption of input preference orderings or rankings for each individual is a tacit assumption of equal utility scales for each individual equivalent to the “one man, one vote” principle. With the assumption that individual orderings represent equally spaced utilities, we can convert orderings or rankings into ratings. This may or may not be a very accurate representation of the underlying utilities, but it's the best information available if only individual orderings are known. These ratings can then be used as inputs to the OTM mechanism.

The available information for rankings is of the form *aRbRcRd*... . For the system considered here and without loss of generality, any scale with any end points can be used for this conversion procedure as long as the preference ratings are equally spaced. For instance, we can choose the real line between "0" and "+1". We let the top ranked candidate be placed at "+1" and the lowest ranked candidate be placed at "0". The other candidates then would be equally spaced on the scale. The OTM information processing system will then output approval style positive choices for those candidates represented by utilities above the optimal threshold and zero choices for those candidates represented by utilities below the optimal threshold for each individual. As we have shown, any affine linear transformation of an individual's utility scale will not change the results of the OTM mechanism. The outputs are in the form of integers and represent the votes or choices for each alternative or candidate. Thus individual inputs can be in the form of rankings if utility information is not available. Therefore, the OTM inputs and outputs can both be represented as rankings (orderings) and/or ratings (utilities).

Summary and Conclusions

We have proved that the OTM mechanism satisfies Arrow's five rational and normative conditions. It has been shown that social choice is possible thus replacing both Arrow's and Gibbard-Satterthwaite's impossibility theorems which are devoid of the inherently probabilistic nature of voting/choosing methods. Their results apply to certain deterministic mathematical structures and were not extended to the more realistic probabilistic case considered here. We have developed a completely new concept, the Optimal Threshold Mechanism (OTM), which accepts Arrow's and Gibbard-Satterthwaite's conditions and yet produces actual possible results. Furthermore, since we deal with utilitarian rather than preference ordering information, the results manifest an upgraded and more robust version of Arrow's normative conditions. Utilitarian satisfaction is also measurable both at the individual and social levels after the choosing process occurs. The OTM system accepts individual utilitarian style preference ratings as inputs and outputs approval style social choice preference rankings. It processes the inputs in such a way as to maximize the expected utility of the social choice for each individual chooser based on their input alone. This is done by setting an optimal threshold in the input utilitarian data of each individual chooser and outputting "+1" approval style choices for those candidates above threshold and "0" approval style choices for those candidates below threshold. Thus the input data is converted into approval style outputs which are then summed over all choosers. This produces social choice rankings for all of the alternatives. The optimal threshold resolves the issue in approval voting of how to accurately divide the candidates into two groups. Since the OTM system converts utilitarian style inputs to approval style outputs, OTM is a utilitarian approval hybrid (UAV) system. Our analysis covers the cases in which no polling or probability information is known, and also when probability or polling information is known.

The issue of interpersonal comparisons is moot because any affine linear transformation of an individual's utility scale will produce the same results when processed by the OTM system. If inputs are specified as preference rankings rather than ratings, the rankings can be converted to utility style ratings which can then be processed by the OTM system. The outputs which are in the form of social rankings can also be converted back to ratings because the underlying utility information for each individual chooser is known. The utility of the social choice can be computed for each individual and for society as a whole.

The OTM system will produce the utilitarian winner(s), that is the winner(s) that maximize social utility. It has been shown by other writers (Lehtinen,2015: p.35) that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior ". In the OTM system considered here, the strategy is implemented by the mechanism itself so that individual choosers have no incentive other than to input honest and sincere choices.

Finally, a maximin or leximin condition can be applied to the outcome which raises the utility levels of those with the least at the expense of the diminution of maximal social utility.

Arrow's main conclusion has been known since 1785 from the work of the Marquis de Condorcet, but Arrow attempted to elaborate and recast the paradox of voting as a proof that any kind of rational system which purports to determine the public good instead leads to a dictatorship. The work presented here proves that direct political and economic democracy do in fact have a sound scientific basis and that rational and normative social choice is indeed possible.

Appendix A

The OTSC Mechanism Satisfies Arrow's Five Conditions

Arrow's five rational and normative conditions are

1) Unrestricted domain.

2) Positive Association of Individual and Social Values

3) Independence of Irrelevant Alternatives (IIA)

4) Citizens' Sovereignty

5) Non-dictatorship

Lemma 1

xR_jy iff $u_{xj} \geq u_{yj}$, by definition

xR_jy iff $b_{xj} \geq b_{yj}$, by definition

xP_jy iff $u_{xj} > u_{yj}$, by definition

xP_jy iff $b_{xj} > b_{yj}$, by definition

Lemma 2

With reference to algorithm **σ,** $b_{ij} = 1$ iff $u_{ij} \geq t_j^*$. $b_{ij} = 0$ iff $u_{ij} < t_j^*$.

$b_{xj} = 1 \wedge b_{yj} = 0$ iff $u_{xj} \geq t_j^* \wedge u_{yj} < t_j^*$

$b_{xj} = 0 \wedge b_{yj} = 0$ iff $u_{xj} \wedge u_{yj} < t_j^*$

$b_{xj} = 1 \wedge b_{yj} = 1$ iff $u_{xj} \wedge u_{yj} \geq t_j^*$

$b_{xj} = 0, b_{yj} = 1$ iff $u_{xj} < t_j^* \wedge u_{yj} \geq t_j^*$

b_{xj} = AV style vote for x in U_j

b'_{xj} = AV style vote for x in U'_j

$b_{y'j}$ = AV style vote for y' in U_j

$b'_{y'j}$ = AV style vote for y' in U'_j

Proof of Condition 1: Unrestricted Domain

By assumption any alternative, c_{ij}, can be given any utility rating, $u_{ij} \in \mathbb{R}^+ \ \forall$ i,j. Neutrality is assumed with respect to the alternatives. The OTSC mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of alternatives, C, $R[\psi(c_1), ..., \psi(c_n)] = \psi[R(c_1, ..., c_n)]$. According to Fleurbaey and Hammond (2004: p.37) Cardinal Full Comparability (CFC) asserts that an affine linear transformation so that $0 \leq u_{ij} \leq 1$, which is the assumed input to the OTSC system, will not change the results. Any affine linear transformation of a chooser's utility profile will yield the same social choice results since the optimal threshold is a function of n_j*. Without loss of generality, the OTSC system will preprocess the input utility profile and perform the affine linear transformation.

Lemma 3

For the purposes of the proof of Condition 2, we change our notation to the notation Arrow uses. x, y, x' and y' become specific to Arrow's statement of the problem and not the same as the notation used previously in this paper.

Let u_{1j}, u_{2j}, ... u_{nj} and u'_{1j}, u'_{2j}, ... u'_{nj} be two sets of utility profiles corresponding to the two sets of ordering relations, R_1,..., R_j, ..., R_n and R'_1,..., R'_j, ..., R'_n with $u_{1j} \geq u_{2j} \geq ... \geq$

$u_{ij} \geq \ldots \geq u_{nj}$ and

$u'_{1j} \geq u'_{2j} \geq \ldots \geq u'_{ij} \geq \ldots \geq u'_{nj}$. In terms of the OTSC mechanism we have $c_{1j}\,\mathrm{R}c_{2j}\,\mathrm{R}\;c_{3j}\,\mathrm{R}c_{4j}$,

...,

$c_{xj}\,\mathrm{R}c_{x+1j}$,...,$c_{yj}\,\mathrm{R}c_{y+1j}$,..., $c_{n-1j}\mathrm{R}c_{nj}$ and $c'_{1j}\,\mathrm{R}'c'_{2j}\,\mathrm{R}'c'_{3j}\,\mathrm{R}'c'_{4j}$,...,$c'_{x'j}\,\mathrm{R}'c'_{x'+1j}$,...,$c'_{y'j}\,\mathrm{R}'c'_{y'+1j}$,...,$c'_{n-1j}\mathrm{R}'c'_{nj}$. Let c_{xj} = x and c_{yj} = y; Let $c'_{x'j}$ = x' and $c'_{y'j}$ = y'.

Proof of Condition 2: Positive Association of Social and Individual Values

Statement of Condition 2: Let R_1,..., R_j, ..., R_n and R'_1,..., R'_j, ..., R'_n be two sets of individual ordering relations, R and R' the corresponding social orderings, and P and P' the corresponding social preference relations. Suppose that for each *j* the two individual ordering relations are connected in the following ways: for x' and y' distinct from a given alternative x, $x'R'_jy'$ if and only if $x'R_jy'$; for all y', xR_jy' implies xR'_jy'; for all y', xP_jy' implies xP'_jy'. Then if xPy', xP'y'.

Proof:

By assumption, $x'R'_jy'$ if and only if $x'R_jy'$

∴ By Lemma 1, $b'_{x'j} \geq b_{y'j}$ iff $b_{x'j} \geq b_{y'j}$

By Lemma 2,

$b'_{x'j} = 1 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 1 \wedge b_{y'j} = 0$

$b'_{x'j} = 1 \wedge b'_{y'j} = 1$ iff $b_{x'j} = 1 \wedge b_{y'j} = 1$

$b'_{x'j} = 0 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 0 \wedge b_{y'j} = 0$

$\therefore \rceil (b'_{y'j} = 1 \wedge b'_{x'j} = 0)$ iff $\rceil (b_{y'j} = 1 \wedge b_{x'j} = 0)$

$$\therefore \sum_j b_{x'j} = \sum_j b'_{x'j} \;\forall\, x', j$$

$$\therefore \sum_j b_{y'j} = \sum_j b'_{y'j} \;\forall\, y', j$$

$$\mathbf{xP'y \text{ iff } \sum_j b_{xj} > \sum_j b_{y'j} \;\forall\, y', j}$$

$$\mathbf{xP'y' \text{ iff } \sum_j b'_{xj} > \sum_j b'_{y'j} \;\forall\, y', j}$$

By assumption, If $xP_j\, y'$, then $xP'_j y'$ $\forall\, y', j$

$$\mathbf{\text{If } b_{xj} > b_{y'j} \text{ then } b'_{xj} > b'_{y'j}}$$

$$\mathbf{\text{If } \left(\sum_j b_{xj} > \sum_j b_{y'j}\right) \text{ then } \left(\sum_j b'_{xj} > \sum_j b'_{y'j}\right)}$$

$\therefore$ If xPy' then xP'y' Q.E.D.

Discussion:

Condition (2) is satisfied because raising some alternative's utility, u_{ij}, in an individual's utilitarian style input from just under to just above optimal threshold will result in that alternative's receiving one more approval style choice, b_{ij}, in the final summation, *X*. This would raise the social choice result by one for that alternative potentially putting that alternative in the winning set and/or changing the ordering in the set, *Y*. Similarly, lowering a candidate's rating in some individual's utility scale might eliminate that alternative from the winning set or change the ordering of the set, *Y.*

Proof of Condition 3: Independence of Irrelevant Alternatives (IIA)

We state Arrow's Condition 3 as follows:

Let $R_1,\dots, R_j ,\dots, R_n$ and $R'_1,\dots, R'_j ,\dots, R'_n$ be two sets of individual orderings and let C(S) and C'(S) be the corresponding social choice functions. If, for all individuals j and all x and y in a given environment S, xR_jy if and only if xR'_jy, then C(S) and C'(S) are the same (independence or irrelevant alternatives).

Proof:

Let S = {x,y}

To prove: $C(S) = C'(S) = x \ \forall\ x,y$

By assumption, xR_jy iff $xR'_jy \ \forall\ j$

By Lemmas 1 and 2, $\lceil(b_{xj} = 0 \wedge b_{yj} =1)$ iff $\lceil(b'_{xj} = 0 \wedge b'_{yj} =1) \ \forall\ x,y,j$

$\therefore \lceil yPx$ iff $\lceil yP'x \ \forall\ x,y$

$\therefore\ C(S) = C'(S) = x$

Q.E.D.

Discussion:

Utilitarian style sincere ratings for each candidate are assumed to be independent of each other regardless of the composition of the alternative set. (Hillinger, 2004: p. 3), "A cardinal number assigned to an object indicates its place on a scale that is independent of other objects." So if an individual rates a candidate at a particular rating on their utility scale, and then another candidate enters or leaves the race, it is assumed that the first candidate will still be rated the same. A candidate's dropping out or entering the race is assumed not to change an individual's sincere ratings for the other candidates.

Now consider the case in which, after the election occurs, a candidate dies or drops out. Arrow (1951: p. 26) states : “Suppose that an election is held, with a certain number of candidates in the field, each individual filing his list of preferences, and then one of the candidates dies. Surely the social choice should be made by taking each of the individual's preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining candidates in going through the procedure of determining a winner.” Arrow implies that the voting has already occurred, but the final determination of the winner(s) has not been made. If this were the case, the OTM system would blot out the dead candidate's rating from all of the individual rating scales, recompute all the individual thresholds, and recompute the ordered outcome, *Y*, and the winning set, *W*. Therefore, the dead candidate is not irrelevant, just not included in the final computation.

Now consider the case in which a new candidate enters the race after the balloting has occurred but before the election results have been published. The added utility rating for that candidate would be uploaded to the OTM system by each individual chooser after the utilities for the other candidates had presumably already been submitted, and the results had already been computed. The OTM system would then recompute the individual thresholds including the added candidate's utility ratings and the final social choice results would then be recomputed. The individual choosers would not have an incentive to rate the added candidate insincerely knowing that the OTM system would give them the strategically best outcome based on the complete list of submitted utilities. Therefore, candidate add-ons would not incentivize any individual chooser to choose insincerely. Furthermore, compliance with IIA is satisfied for add-ons since ratings for two candidates at a time could be uploaded for each individual chooser with thresholds recomputed at each step or as a final step thus demonstrating that the social choice can be arrived at by pairwise comparisons which Arrow's IIA demands.

Condition 4: The Social Welfare Function Is Not imposed.

The ouput of the OTM system is solely a function of the unrestricted inputs by assumption. There are no alternatives x and y such that xRy regardless of voter inputs. The OTM system is neutral and anonymous. It treats all citizens and alternatives the same. All permutations of V, the set of voters, and C, the set of candidates, are allowed. Permutations of voters or candidates do not change the results. The OTM mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of voters, V, $R[\psi(v_1), ..., \psi(v_n)] = \psi[R(v_1, ..., v_n)]$.

Condition 5: The Social Welfare Function Is Not To Be Dictorial

For the OTM system $C_j = \{c_{1j}, c_{2j}, \ldots c_{nj}\}$. All permutations of c_{ij} are allowed, $\forall$ i,j. Condition (5) is satisfied since the winning set is based only on individual inputs which are all treated equally. There is no voter/consumer *j* such that xRy iff xR_jy. Therefore, the OTM mechanism satisfies all five of arrow's rational and normative conditions. Q.E.D.

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