Utilitarian Social Choice and Voting

by

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Abstract

This paper develops a utilitarian version of social choice with application to voting for both single and multiple winner elections. We prove that it satisfies Arrow's rational and normative conditions. The issue of interpersonal comparison of utilities is dealt with by equalizing all inputs to the system after they have been formulated in a utilitarian manner. Most voting systems are plagued by the issue of strategy whereby voters can get a better result for themselves by formulating their input insincerely. We devise a way to elicit sincere information from the voters. This system selects the utilitarian winner(s), the one that maximizes social utility. This makes it possible to implement a maximin provision for representative bodies such that a modification of the results can assure the least well off a minimum level of utility in the results of the election. This can be accomplished in such a way that, subject to this condition, social utility is the highest possible.

Introduction

Kenneth Arrow (1951) invented the field of social choice. Arrow sets up the problem so that each individual voter ranks all candidates and then society is required to come up with an ordering that is best according to his stated criteria.
Arrow states (1951: p. 11-12) “In the theory of consumer's choice each alternative would be a commodity bundle; ... in welfare economics, each alternative would be a distribution of commodities and labor requirements. … in the theory of elections, the alternatives are candidates.” In today's economy rather than an alternative being a distribution of commodities and labor requirements, it might instead be cash payments and labor requirements. In this paper we will deal with the theory of elections; in a companion paper we deal with the welfare economic implications. Social choice is a general system that applies to both political and economic systems, any system in which individual inputs can be amalgamated into an output social result.

Shortly after he had proposed the field of social choice, Arrow went on to prove it was impossible. There was no way to amalgamate individual decisions into a social decision if certain rational and normative conditions are met. Arrow's result, formerly called the *paradox of voting*, was first discovered by the Marquis de Condorcet (1785). Condorcet's paradox showed that majority preferences can become intransitive when there are three or more alternatives. Arrow mathematized Condorcet's insight. Later Gibbard and Satterthwaite proved that any such amalgamation of individual preferences, in which there is no advantage to any individual to use strategy in order to get a better result for themselves, is also impossible.

Utilitarianism was founded by Jeremy Bentham (1789), an English philosopher, jurist, and social reformer. Bentham defined as the "fundamental axiom" of his philosophy the principle that "it is the greatest happiness of the greatest number that is the measure of right and wrong." The word 'utility' is a synonym for the word 'happiness'. Since in general it's not possible to maximize two variables simultaneously, the method described in this paper will demonstrate a mechanism which obtains just the greatest social utility regardless of the number. By using utilitarian ratings instead of the rankings of candidates that Arrow used, social choice is indeed possible.

In the debate among principles of distributive ethics, two of the main contenders are contractualism as represented by John Rawls (2001), and utilitarianism as represented by John Harsanyi (1977), Amartya Sen (2017), Hun Chung (2023) and others. T.M. Scanlon (1982) writes, "Contractualism has been proposed as the alternative to utilitarianism before, notably by John Rawls in *A Theory of Justice*." This paper represents a synthesis of contractualism and utilitarianismin in that a Rawlsian maximin condition can be combined with the utilitarian results.

Arrow assumed ordinal voting inputs of the following form. Let $C = \{c_{1j}, c_{2j}, ..., c_{nj}\}$ be an ordered set of candidates of size n such that $c_{1j}R_jc_{2j} ... c_{(n-1)j} R_jc_{nj}$ for $1 \leq j \leq q$ where R_j means "is preferred or indifferent to". q is the total number of voters. Each voter, j, orders the candidates, i, according to their preferences. A utilitarian approach also considers the utilities associated with each candidate. Let $U_j = \{u_{1j}, u_{2j}, ... u_{nj}\}$ be a set of utilities of size n, with $u_{1j} \geq u_{2j} \geq ... \geq u_{nj}$. u_{ij} is the utility of voter j for candidate c_{ij}. U_j is called a utility profile.

Claude Hillinger (2005: pp. 295-321) has made the case for utilitarian voting:

> "There is, however, another branch of collective choice theory, namely utilitarian collective choice, that, instead of fiddling with Arrow's axioms,

challenges the very framework within which those axioms are expressed. Arrow's framework is ordinal in the sense that it assumes that only the information provided by individual orderings over the alternatives are relevant for the determination of a social ordering. Utilitarian collective choice assumes that individual preferences are given as cardinal numbers; social preference is defined as the sum of these numbers."

The social choice mechanism discussed here differs from Hillinger's statement in that there is a unique transformation done by the voting system or mechanism itself for each voter from their cardinal inputs to their ordinal contribution to the social choice output.

According to Warren Smith (2023): "Kenneth Arrow assumed that a voting method ("social choice function") must be based on *ranking* of options by voters, then analyzed them in terms of their logical *properties*. This, absurdly, makes [score voting] "not a voting method." The truth: any algorithm that inputs votes and outputs winners is a "voting method"; we should allow "votes" to be any information packet whatever." Range or score voting is a subset of utilitarian voting in which there are a discrete number of levels that can be used to rate candidates. In utilitarian voting, utility can be measured using the set of non-negative points u_{ij} on the real line, $R_{\geq 0}$. It's up to the individual voter where to place the points, including the end points, corresponding to the utilities of each candidate in the candidate set.

Interpersonal Comparisons of Utility

Arrow (1951: p. 10) dwells on the fact that individual utility scales are not compatible. He compares them with the measurement of temperature which is based on arbitrary units and the arbitrary terminal points of freezing and boiling for the Celsius scale and completely different end points for the Fahrenheit scale.

> "Even if, for some reason, we should admit the measurability of utility for an individual, there still remains the question of aggregating the individual utilities. At best, it is contended that, for an individual, their utility function is uniquely determined up to a linear transformation; we must still choose one out of the infinite family of indicators to represent the individual, and the values of the aggregate (say a sum) are dependent on how the choice is made for each individual. In general, there seems to be no method intrinsic to utility measurement which will make the choice compatible."

Arrow's analysis is based on how an outside observer would select an indicator to represent each individual. Our analysis is based on how each individual, theirself, would choose their own indicator. We take into account that each individual has a unique utility profile. There is no need to (Arrow: p. 12) “choose one out of the infinite family of indicators to represent the individual.” Each individual chooses their own indicator.

The problem with summing up the utilities of different individuals for each candidate is that, although the ratings may have meaning for each individual, they don't have any meaning in common for the different individuals involved. For instance, a rating of 1.5 for the utility of candidate A by voter 1 might have a different meaning from a rating of 1.5 for candidate A from voter 2. The way out of this dilemma has been presaged by Amartya Sen. Sen's (2017) Cardinal Non-Comparability condition (CNC) states that for all $U_j, U'_j \in \mathbb{R}$, one has $f(U_j) = f(U'_j)$ whenever for all $j \in V$, there are real constants κ_j

and ν_j, with each $\nu_j > 0$, such that $U'_j \equiv \kappa_j + \nu_j U_j$. If we apply an affine linear transformation to the utility scales of all voters such that they are converted to utilities between *0* and *1*, for instance, we will have equalized all voting inputs while retaining utilitarian information. Equalization of inputs is standard for all voting systems in the sense of "one man, one vote". Therefore, we will not be concerned further about interpersonally comparing utilities. This transformation may be part of the voting system or mechanism itself, or for simplification purposes, may be required of each voter so that each voter expresses their preferences as real numbers between *0* and *1*.

Strategy

It is well known that many voting systems are vulnerable to the use of strategy by individual voters. The inventor of one of these systems, the Borda count, said famously, "My scheme is intended only for honest men." (Black, D. 1958. p. 182) However, strategy may actually have some desirable consequences. "Under the utilitarian evaluation of voting outcomes, what has been thought of as a major disadvantage of the Borda rule turns out to be an argument for it." (Lehtinen, 2007**)**

Gibbard and Satterthwaite concurred with Arrow and proved that any social choice mechanism that was strategy proof was also impossible. Gibbard (1973: p. 587) states: “An individual 'manipulates' the choosing scheme if, by misrepresenting his preferences, he secures an outcome he prefers to the 'honest' outcome - the choice the community would make if he expressed his true preferences." Satterthwaite (1975: p.188) showed that the requirement for voting procedures of strategyproofness and Arrow’s requirements for social welfare functions are equivalent: "a one-to-one correspondence exists between every strategy-proof voting procedure and every social welfare function satisfying Arrow’s four requirements."

Jackson (2001: p. 2) states: "Often, one thinks of the desired outcomes as the given and analyzes whether there exist game forms for which the strategic properties induce individuals to (always) choose actions that lead to the desired outcomes." In social choice theory, a strategy-proof (or non-manipulable) voting mechanism is one where no voter can achieve a better outcome for themselves by misrepresenting their true preferences. We consider a mechanism for which the strategic properties *do* induce individuals to choose actions that lead to the desired outcome – a possible social choice– while disincentivizing them from choosing strategically as individuals.

Gibbard's (1973: p. 590) results were based only on the possibility that someone could use strategy if they were astute enough to stumble on a way to do so. "Note that to call a voting scheme manipulable is not to say that, given the actual circumstances, someone is really in a position to manipulate it." Only the possibility exists in an elaborate deterministic mathematical structure. Gibbard doesn't assume that there is any formularizable or identifiable strategy that an individual voter could use to manipulate the system. Other writers have pointed out this difficulty: (Meir et. al.: p. 149) "In other words, computational complexity may be an obstacle that prevents strategic behavior." We analyze a situation in which an actual identifiable strategy exists which can be known both to the individual voter and to the mechanism, which amalgamates or processes the votes, itself. If the mechanism does the strategizing for each individual, there is no incentive for the individual to do so, and sincere information can be elicited from the voters.

Gibbard's and Satterthwaite's analysis is deterministic while the problem of manipulability is inherently statistical. In an actual election it would be impossible for a voter to know the ideal strategy unless they knew how every other voter was going to vote. We assume no deterministic knowledge of how other voters will vote. We identify the best strategy for a particular voter if other voters are assumed to vote completely randomly, and the individual voter has no knowledge of how other voters will vote. If the voting system which amalgamates the voters' inputs applies the strategy, the voters themselves are disincentivized from doing so, and sincere voting information can be elicited from the voters. Other writers (Cranor,1996; LeGrand, 2008) have also sought to develop systems such as Declared-Strategy Voting which attempts to "elicit more sincere preferences from voters ... to find a winning alternative in such a way that voters would be unlikely to gain a superior result by submitting insincere preferences."

Lehtinen (2015: p.35) has shown that "strategic behavior increases the frequency with which the utilitarian winner is chosen compared to sincere behavior ". He defines the *utilitarian winner* as the one that maximizes the social utility of the social choice. After the social choice which maximizes utility has been found, a maximin condition can be applied which raises the utility levels of those with the least utility to a minimum acceptable level while diminishing overall social utility by the least amount. This can only be done if sincere information has been elicited from the voters. Otherwise, a maximin condition makes no sense (garbage in, garbage out). Therefore, the mechanism described in this paper should accomplish three things: (1) sincere voting behavior on the part of individuals; (2) increased selection of the utilitarian winner or winners and (3) the possibility of adding a maximin provision while diminishing social utility by the least mount. While Lehtinen abandons the Arrow and Gibbard-Sattherwaite conditions in the interests of increased social utility, strategyproofness is not violated if the mechanism, which amalgamates the individual voting information, itself applies the strategy instead of the individual voters doing so. This introduces another layer of complexity to the voting system, but other voting systems such as IRV (instant runoff voting) are also more complex than those which simply count ballots.

There has been much criticism of various voting systems in the literature because of their susceptibility to strategy. According to N. Tideman (2023), "The use of range [also know as score] voting is particularly unsatisfying if it is expected that some voters will abide by the norm that scores are to be non-strategic while others will not, for in that case the power to affect the outcome will be much greater for those who do not abide by the norm than for those who do." Although range and utilitarian voting are "particularly unsatisfying" if some voters vote strategically and some do not, when the mechanism or system which amalgamates the votes itself applies the strategy for every voter, then this issue is equalized among voters. Tideman's (2023) evaluation of approval voting is even worse. "It is quite easy to imagine a voter approaching the task of scoring or ranking a set of candidates as something to be done by consulting one's truthful evaluations and eschewing any possible gain from strategic voting. With approval voting, on the other hand, it is readily imaginable that a voter will see no possibility of a non-strategic vote."

Most of the proposed voting systems (Constitutional Political Economy, 2023) are concerned with being resistant to strategizing by the voters. For the system presented here the optimal strategy can be known both to individual voters and to the mechanism that amalgamates the voting information itself. If the mechanism applies the optimal strategy to each individual vote, voters are disincentivized from strategizing individually. Therefore, sincere voting information can be elicited. To vote otherwise would give the voter possibly a worse outcome than if they had voted sincerely. Since the voting mechanism applies the optimal strategy to each vote, the social utility of the outcome should also be increased according to Lehtinen (2015: p.35). The eliciting of sincere information from the voters then makes it possible in a multiwinner election for the mechanism to apply a Rawlsian maximin condition to the outcome of the election which results in the raising of utilities of the least well off to some minimum level while lowering the overall social utility by the least amount. The complexity of this process would require an AI solution especially for large numbers of members of a legislature.

Utilitarian Approval Voting

After the original placement of utilities on the real line, $R_{\geq 0}$, each voter's expression of utilities is equalized by means of an affine linear transformation to the real line between *0* and *1*. Then using strategy a threshold is placed such that all those above threshold get an approval vote of *1* and all those below get an approval vote of *0*. This turns the utilitarian vote into an approval vote. The question is where to place the threshold so as to maximize the voter's outcome in the election results. For example, let's say there are two candidates, A and B, and the voters vote utilitarian style. We will assume that a particular voter has a utility of .8 for candidate A and .4 for candidate B. Obviously, this voter prefers that candidate A and not candidate B becomes the winner of the election. Rather than submit his sincere utilities to the voting system, this voter can vote strategically giving candidate A a utility of *1* and candidate B a utility of *0*. If there are several candidates in the race, a voter could list his sincere utilities for each candidate as real numbers between *0* and *1*. Then voting strategically, they would elevate some candidates to a vote of *1* and lower others to a vote of *0*. Voting strategically with utilitarian voting (UV) devolves into approval voting (AV). The question is where to draw the line so that an individual's utility in the outcome of the election is maximized.

The method constructed in this paper takes input information from the individual voters in the form of preference ratings over each candidate and outputs information in the form of complete social preference rankings of the candidates from which social and individual ratings can be derived. Since the system itself applies the optimal strategy to each vote cast, we can assume that sincere voter ratings have been submitted by the voters. From the social preference rankings, an unordered winning set, *W,* of size *m,* $m \geq 1$, is constructed consisting of those candidates with the top *m* rankings. The utility of the winning set for each voter, which is the summation of their utilities over each candidate in the winning set, can be computed since we know from the individual inputs how each voter sincerely rated each candidate. Summing utilities over all voters gives the social utility of the winning set. The winning set for $m > 1$ would be the members of a legislative body or in the case of $m = 1$, a President.

In order to overcome the Gibbard-Sattertwaite theorems, which maintain that every voting system for which an individual voter can use strategy to improve the outcome for themselves violates Arrow's conditions, we choose a social choice mechanism which itself implements the optimal strategy for each individual assuming that that strategy consists of each individual's voting in such a way as to maximize the expected utility of the winning set for themselves. We assume a completely random distribution of voter preferences and assume that each voter has no knowledge of the utility profiles of other voters. The optimal strategy can be known both to the individual voter and to the mechanism which amalgamates the votes itself. If the mechanism applies the optimal strategy to each voter's input, then the individual voter is disincentivized from doing so and *is* incentivized to submit their sincere utilities.

Each candidate is rated by assigning to them a real number between zero and one. The utility profile, U_j, consists of this set of ratings where j signifies the specific individual voter. The mechanism described herein involves the placing of an individualized threshold in the monotonically increasing and unrestricted utility profile which is submitted by each voter. Each candidate above this threshold is given an approval style vote of 1, and each candidate below threshold is given an approval style vote of 0. This strategy can be seen as the extension of the strategy when there are only two candidates. The threshold is placed such that the expected utility of the set of candidates above threshold is a maximum.

As the threshold increases, there are less candidates above threshold, the average utility rating of the set of candidates above threshold increases, and the probability of selecting randomly any particular candidate in this set decreases. Conversely, as the threshold decreases, the number of candidates above threshold and the probability of random selection of one of them increases while the average utility rating of that set of candidates decreases. The mechanism we explore here chooses the optimal threshold, individualized for each voter, to be just under that utility such that the expected value of their utility in the winning set, *W*, is a maximum. The formal description of parameters for this system is found in Appendix B.

The Optimal Threshold Mechanism

The Optimal Threshold Mechanism (OTM) Information Processing System can be modeled as follows:

Figure 1

We focus now on one particular voter called the focal voter. While Brams and Fishburn (1983: p. 73) "presume that voters' preferences are more or less evenly distributed over the different preference orders for the ... candidates," we determine the best way for a voter with a particular preference order, in this case a utility profile, U_j, to vote. We analyze the focal voter's efficacy in changing the election results using strategy due to their input alone. The strategy involves separating the candidates into two dichotomous sets by placing a threshold in the focal voter's set of utilities such that approval style votes of *1* are cast for utilities above threshold and approval style votes of *0* are cast for utilities below that threshold. With reference to Appendix B,

$B_j = \{b_{1j}, b_{2j}, ..., ,b_{nj}\}$ is a set of approval style votes in order of the j^{th} voter's candidate preferences for candidate i. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$.

We model the situation as a ball and urn problem consisting of n black and white balls representing the candidates. We identify the white balls with candidates above threshold and black balls with candidates below threshold. Let n_{aij} be the number of candidates above threshold. m balls are chosen randomly out of the urn without replacement and placed in the winning set, W. The probability, p, of k above threshold candidates being in the winning set due to chance alone is given by the hypergeometric function:

$$p(k) = \frac{\binom{n_{aij}}{k}\binom{n-n_{aij}}{m-k}}{\binom{n}{m}}$$

Exactly which white ball (associated with a particular candidate) is picked is not known. However, the average utility of above threshold candidates, u_{aij}/n_{aij}, can be calculated.

Let u_{Wj} be a random variable which represents the expected value of the average utility of above threshold candidates being in the winning set for voter j at threshold t_{ij}:

$$E_{t_{ij}}(u_{Wj}) = \sum_{k=1}^{s} \left\{ \left[\frac{\binom{n_{aij}}{k}\binom{n-n_{aij}}{m-k}}{\binom{n}{m}} \right] \left[\frac{u_{aij}}{n_{aij}} \right] \right\}$$

where $s = min\{ m, n_{aij}\}$ and $0 \leq u_{Wj} \leq 1, \forall j$.

We now perform a thought experiment in which we calculate the value of expected utility for every threshold, $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1$. At every threshold balls are randomly withdrawn from the urn and placed in the winning set. Let t_j^* be the optimal threshold which is the threshold which results in the maximization of the expected value of average utility, $E_{t_{ij}}(u_{Wj})$, for voter *j*. Therefore, the optimal threshold is the one for which the following equation holds:

$$E_{t_j^*}(u_{Wj}) = \max\{E_{t_{ij}}(u_{Wj})\}$$

The set of candidates above optimal threshold is called the optimal set, O_j. $O_j = B_j$ such that $b_{ij} = 1$. As the threshold is decreased from t_j^*, the average utility of the above threshold set for voter *j* decreases because there are more above threshold utilities with lower values of utility under consideration, and the probability of an above threshold candidate being in the winning set increases. As the threshold is increased from t_j^*, the probability of an above threshold candidate being in the winning set decreases, and the average utility of the set of candidates above threshold increases. Candidates whose utilities are greater than the optimal threshold, t_j^*, will be given the maximum AV vote of *1*, and candidates whose utilities are less than t_j^* will be given the minimum AV vote of $0 \; \forall j$. The individual voter's strategy, which is the same as the mechanism's strategy, is to give a one vote boost to candidates above threshold, which belong to the set for which the voter has the greatest expected average utility in the outcome of the election.

If the ball and urn experiment were to be performed on each member of the electorate as a whole minus the focal voter, the total number of white balls representing AV votes for each candidate could be added up. With the addition of the focal voter's input, there is a finite probability that one or more candidates would be elevated to the winning set resulting in a tie or ties with a candidate already in the winning set. The focal voter's AV votes could potentially determine the constitution of the winning set if a candidate's being in the winning set can be determined by single votes after all other voters have cast their ballots and ties are resolved randomly.

The OTM mechanism uses the above analysis to optimize each individual's vote so that the voters are disincentivized from choosing insincerely. It addresses Gibbard-Satterthwaite's concerns about strategic voting by individuals while meeting Arrow's rational and normative conditions as proven in Appendix A. It even upgrades Arrow's normative conditions since more finely tuned cardinal input information is used while Arrow's analysis only involved less precise ordinal information. Moreover, the welfare or utilitarian results for each individual and for society as a whole are *measurable*. The key is that individuals are disincentivized from voting insincerely because the OTM system strategizes for them. The optimal strategy maximizes the expected value of utility of the winning set, W, for each voter based on their vote alone.

The assumption of utility maximizing is made by other writers (Lehtinen, 2008: pp. 688-704): "Under strategic behaviour voters are assumed to maximise expected utility ... ". The voter's input is the ordered set of candidates C_j and the associated ordered set of utilities U_j. The output is the set Y consisting of the ordered set of all candidates by vote totals from which is derived the winning set, W, which is unordered and consists of $m < n$ candidates. It is assumed that each individual voter specifies an unrestricted, utilitarian style input profile, U_j, which represents their sincere utility ratings for candidates in the set, C.

The vote count proceeds by the following algorithm, **σ**:

```
σ: for z = 1, n
     x_z = 0
end z (initializes X)
     for j = 1,q
        for i = 1, n
             b_ij = 0
             if {u_ij ≥ t_j*}  then
                 b_ij = 1
```

$x_i = x_i + 1$

end i

end j

end **σ**

Let ${}^{A}u_j$ be the utility of the winning set, *W*, for voter *j* post-election, and ${}^{A}u$ be the social utility of the winning set for all voters i.e. the utility of the social choice. With reference to Appendix B:

$${}^{A}u_j = \sum_{i=1}^{m} \eta_j \chi_j \tau_i^{-1} \alpha_i^{-1} \beta_i^{-1} (w_i) \qquad {}^{A}\mathbf{u} = \sum_{j=1}^{q} {}^{A}\mathbf{u}_j$$

Utilitarian Winners and Minimax Conditions

The Optimal Choice Mechanism (OCM) considered here chooses the expected utilitarian winner(s) in the sense that it maximizes the expected value of utility for the winner(s) of the election over all voters. Following is a proof by contradiction.

Consider the winning set: $W = \{w_1, w_2, ..., w_m\}$. Assume that if one of the members of the winning set is replaced by another candidate, c_{ij}, the expected utility of the winning set will be greater. $W = \{w_1, w_2, ..., c_{ij}, ..., w_{m-1}\}$ This means that some voter, *j*'s, utility for candidate, c_{ij}, will go from below threshold to above threshold so that c_{ij} will have one more vote and could become part of the winning set. This means lowering voter *j's* threshold in order to give c_{ij} one more vote. Therefore, voter *j's* expected utility in the outcome of the election is not maximized because the optimal threshold was not chosen, and the total expected social utility is less than if one of the members of the winning set had not been replaced with c_{ij}. This is a contradiction. Therefore, the expected utility of the winning set, $W = \{w_1, w_2, ..., w_m\}$ is maximum and it is the expected utilitarian winner.

After the optimal set has been established and the output utilities computed for each individual, the worst off person or set of persons might have their utility results improved at the expense of a diminution of total social utility. A solution can be found such that the social utility is a maximum given the condition that everyone has at least a utility of u_{min}. Start by substituting the candidate with the highest utility but not in the winning set. Place this candidate in the winning set in place of the candidate in the winning set with the lowest utility and then, if this improves the utilities of the set that was below u_{min} sufficiently, that's a potential minimax solution. Otherwise, try the same replacement sequentially with every other candidate not in the winning set. If there are still some voters with utilities less than u_{min}, substitute the next highest utility candidate not in the winning set for the second lowest candidate in terms of utility in the winning set and recalculate utilities. Try all possible combinations by substituting candidates not in the winning set for candidates in the winning set. For all combinations such that all voters have at least a utility greater than u_{min}, find the one that results in the greatest social utility. That would be the final result. This is a problem which would require the power of AI to solve because of the enormous computing time required especially for a legislative body of hundreds of representatives.

Summary and Conclusions

We have proved that the OTM mechanism satisfies Arrow's five rational and normative conditions. Furthermore, since we deal with utilitarian rather than preference ordering information, the results manifest an upgraded and more robust version of Arrow's normative conditions. The Gibbard-Satterthwaite's impossibility theorems, which state that no social choice is possible which is not susceptible to the use of strategy by the voters, has also been dealt with. Their theorems considered only deterministic data whereas in the real world it is highly unlikely that a particular voter could know how every other voter would vote so that they could apply the best strategy. The concept considered here accepts Arrow's and Gibbard-Satterthwaite's conditions and yet produces actual social choice results that are immune to the use of strategy because the OTM system itself applies the strategy for each individual voter. Therefore, sincere voting information is more likely to be elicited from the voters. Utilitarian satisfaction is also measurable both at the individual and social levels after the voting process occurs. The OTM system accepts individual utilitarian style preference ratings as inputs and outputs approval style social choice preference rankings. It processes the inputs in such a way as to maximize the expected utility of the social choice for each individual voter based on their input alone. This is done by setting an optimal threshold in the input utilitarian data of each individual voter and outputting plus *1* approval style votes for those candidates above threshold and *0* approval style votes for those candidates below threshold. Thus the input utilitarian data is converted into approval style outputs which are then summed over all voters. This produces social choice rankings for all of the candidates. The optimal threshold resolves the issue in approval voting of how to accurately divide the candidates into two groups. Since the OTM system converts utilitarian style inputs to approval style outputs, OTM is a utilitarian approval hybrid (UAV) system. Our analysis covers the case in which no polling or statistical information about how other voters will vote is known.

We deal with the issue of interpersonal comparisons by equalizing all utilitarian inputs. Any utilitarian specification of non-negative real numbers can be converted to values in the range from *0* to *1* by means of an affine linear transformation. This can be done by all voters initially or by the system itself. This makes all voter inputs equivalent in terms of "one man, one vote" similar to any other voting system. Sen's cardinal non-comparability principle illustrates the fact that utilitarian information can be preserved while all voting inputs can be equalized. This does not mean that the resultant values have the same meaning for all participants, but that the principle of "one man-one vote" is preserved.

After the input voting information has been processed by the OTM system, the outputs are in the form of numerical votes which determine the social rankings of the candidates. These rankings can be converted back to ratings because the underlying sincere utility information from each individual voter is known. Since the OTM system itself provided the strategy for each voter, the use of strategy by individual voters has been obviated. This makes it possible to compute the utility of the social choice for each individual and for society as a whole. The OTM system will produce the expected utilitarian winner(s), that is the winner(s) that probabilistically maximize social utility. It has been shown by other writers (Lehtinen,2015: p.35) that "strategic behavior increases the frequency with which the *utilitarian winner* is chosen compared to sincere behavior".

Finally, a maximin condition can be applied to the outcome of the election which raises the utility levels of those with the least utility at the expense of a diminution of social utility after the preliminary election results have been computed. The logic of this is straightforward, and can proceed in a sequential manner. An AI computer solution could find the final result such that every voter has at least a minimal utility while diminishing total social utility by the least amount. Thus in the election of a large representative body, a few seats might be changed from the results that represent the expected utilitarian winner so that every voter has at least a minimum utility in the results of an election.

Appendix A

The OTM Mechanism Satisfies Arrow's Five Conditions

Arrow's five rational and normative conditions are

1) Unrestricted domain.

2) Positive Association of Individual and Social Values

3) Independence of Irrelevant Alternatives (IIA)

4) Citizens' Sovereignty

5) Non-dictatorship

Lemma 1

xR_jy iff $u_{xj} \geq u_{yj}$, by definition

xR_jy iff $b_{xj} \geq b_{yj}$, by definition

xP_jy iff $u_{xj} > u_{yj}$, by definition

xP_jy iff $b_{xj} > b_{yj}$, by definition

Lemma 2

With reference to algorithm **σ,** $b_{ij} = 1$ iff $u_{ij} \geq t_j^*$. $b_{ij} = 0$ iff $u_{ij} < t_j^*$.

$b_{xj} = 1 \wedge b_{yj} = 0$ iff $u_{xj} \geq t_j^* \wedge u_{yj} < t_j^*$

$b_{xj} = 0 \wedge b_{yj} = 0$ iff $u_{xj} \wedge u_{yj} < t_j^*$

$b_{xj} = 1 \wedge b_{yj} = 1$ iff $u_{xj} \wedge u_{yj} \geq t_j^*$

$b_{xj} = 0, b_{yj} = 1$ iff $u_{xj} < t_j^* \wedge u_{yj} \geq t_j^*$

b_{xj} = AV style vote for x in U_j

b'_{xj} = AV style vote for x in U'_j

$b_{y'j}$ = AV style vote for y' in U_j

$b'_{y'j}$ = AV style vote for y' in U'_j

Proof of Condition 1: Unrestricted Domain

By assumption any alternative, c_{ij}, can be given any utility rating, $u_{ij} \in \mathbb{R}^+$ $\forall$ i,j. Neutrality is assumed with respect to the alternatives. The OTSC mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of alternatives, C, $R[\psi(c_1), ..., \psi(c_n)] = \psi[R(c_1, ..., c_n)]$. According to Fleurbaey and Hammond (2004: p.37) Cardinal Full Comparability (CFC) asserts that an affine linear transformation so that $0 \leq u_{ij} \leq 1$, which is the assumed input to the OTSC system, will not change the results. Any affine linear transformation of a chooser's utility profile will yield the same social choice results since the optimal threshold is a function of n_j*. Without loss of generality, the OTSC system will preprocess the input utility profile and perform the affine linear transformation.

Lemma 3

For the purposes of the proof of Condition 2, we change our notation to the notation Arrow uses. x, y, x' and y' become specific to Arrow's statement of the problem and not the same as the notation used previously in this paper.

Let $u_{1j}, u_{2j}, ... u_{nj}$ and $u'_{1j}, u'_{2j}, ... u'_{nj}$ be two sets of utility profiles corresponding to the two sets of ordering relations, $R_1,..., R_j, ..., R_n$ and $R'_1,..., R'_j, ..., R'_n$ with $u_{1j} \geq u_{2j} \geq ... \geq u_{ij} \geq ... \geq u_{nj}$ and $u'_{1j} \geq u'_{2j} \geq ... \geq u'_{ij} \geq ... \geq u'_{nj}$. In terms of the OTSC mechanism we have $c_{1j} R c_{2j} R\, c_{3j} R c_{4j}, ..., c_{xj} R c_{x+1j}, ..., c_{yj} R c_{y+1j}, ..., c_{n-1j} R c_{nj}$ and $c'_{1j} R' c'_{2j} R' c'_{3j} R' c'_{4j}, ..., c'_{x'j} R' c'_{x'+1j}, ..., c'_{y'j} R' c'_{y'+1j}, ..., c'_{n-1j} R' c'_{nj}$. Let c_{xj} = x and c_{yj} = y; Let $c'_{x'j}$ = x' and $c'_{y'j}$ = y'.

Proof of Condition 2: Positive Association of Social and Individual Values

Statement of Condition 2: Let $R_1,\dots, R_j, \dots, R_n$ and $R'_1,\dots, R'_j, \dots, R'_n$ be two sets of individual ordering relations, R and R' the corresponding social orderings, and P and P' the corresponding social preference relations. Suppose that for each j the two individual ordering relations are connected in the following ways: for x' and y' distinct from a given alternative x, $x'R'_jy'$ if and only if $x'R_jy'$; for all y', xR_jy' implies xR'_jy'; for all y', xP_jy' implies xP'_jy'. Then if xPy', xP'y'.

Proof:

By assumption, $x'R'_jy'$ if and only if $x'R_jy'$

$\therefore$ By Lemma 1, $b'_{x'j} \geq b_{y'j}$ iff $b_{x'j} \geq b_{y'j}$

By Lemma 2,

$b'_{x'j} = 1 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 1 \wedge b_{y'j} = 0$

$b'_{x'j} = 1 \wedge b'_{y'j} = 1$ iff $b_{x'j} = 1 \wedge b_{y'j} = 1$

$b'_{x'j} = 0 \wedge b'_{y'j} = 0$ iff $b_{x'j} = 0 \wedge b_{y'j} = 0$

$\therefore \neg(b'_{y'j} = 1 \wedge b'_{x'j} = 0)$ iff $\neg(b_{y'j} = 1 \wedge b_{x'j} = 0)$

$$\therefore \sum_j b_{x'j} = \sum_j b'_{x'j} \ \forall\, x', j$$

$$\therefore \sum_j b_{y'j} = \sum_j b'_{y'j} \ \forall\, y', j$$

$$\mathbf{xP'y \text{ iff } \sum_j b_{xj} > \sum_j b_{y'j} \ \forall\, y', j}$$

$$\mathbf{xP'y' \text{ iff } \sum_j b'_{xj} > \sum_j b'_{y'j} \ \forall\, y', j}$$

By assumption, If $xP_j\,y'$, then xP'_jy' $\forall$ y',j

$$\mathbf{If \left(\sum_j b_{xj} > \sum_j b_{y'j}\right) then \left(\sum_j b'_{xj} > \sum_j b'_{y'j}\right)}$$ [21]

$\therefore$ If xPy' then xP'y' Q.E.D.

Discussion:

Condition (2) is satisfied because raising some alternative's utility, u_{ij}, in an individual's utilitarian style input from just under to just above optimal threshold will result in that alternative's receiving one more approval style choice, b_{ij}, in the final summation, *X*. This would raise the social choice result by one for that alternative potentially putting that alternative in the winning set and/or changing the ordering in the set, *Y*. Similarly, lowering a candidate's rating in some individual's utility scale might eliminate that alternative from the winning set or change the ordering of the set, *Y*.

Proof of Condition 3: Independence of Irrelevant Alternatives (IIA)

We state Arrow's Condition 3 as follows:

Let $R_1,\ldots, R_j ,\ldots, R_n$ and $R'_1,\ldots, R'_j ,\ldots, R'_n$ be two sets of individual orderings and let C(S) and C'(S) be the corresponding social choice functions. If, for all individuals j and all x and y in a given environment S, xR_jy if and only if xR'_jy, then C(S) and C'(S) are the same (independence or irrelevant alternatives).

Proof:

Let S = {x,y}
To prove: $C(S) = C'(S) = x \ \forall\ x,y$

By assumption, xR_jy iff $xR'_jy\ \forall\ j$

By Lemmas 1 and 2, $\ulcorner(b_{xj} = 0 \wedge b_{yj} = 1)$ iff $\ulcorner(b'_{xj} = 0 \wedge b'_{yj} = 1)\ \forall\ x,y,j$

$\therefore \ulcorner yPx$ iff $\ulcorner yP'x\ \forall\ x,y$

$\therefore\ C(S) = C'(S) = x$

Q.E.D.

Discussion:

Utilitarian style sincere ratings for each candidate are assumed to be independent of each other regardless of the composition of the alternative set. (Hillinger, 2004: p. 3), "A cardinal number assigned to an object indicates its place on a scale that is independent of other objects." So if an individual rates a candidate at a particular rating on their utility scale, and then another candidate enters or leaves the race, it is assumed that the first candidate will still be rated the same. A candidate's dropping out or entering the race is assumed not to change an individual's sincere ratings for the other candidates.

Now consider the case in which, after the election occurs, a candidate dies or drops out. Arrow (1951: p. 26) states : “Suppose that an election is held, with a certain number of candidates in the field, each individual filing his list of preferences, and then one of the candidates dies. Surely the social choice should be made by taking each of the individual's preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining candidates in going through the procedure of determining a winner.” Arrow implies that the voting has already occurred, but the final determination of the winner(s) has not been made. If this were the case, the OTM system would blot out the dead candidate's rating from all of the individual rating scales, recompute all the individual thresholds, and recompute the ordered outcome, Y, and the winning set, W. Therefore, the dead candidate is not irrelevant, just not included in the final computation.

Now consider the case in which a new candidate enters the race after the balloting has occurred but before the election results have been published. The added utility rating for that candidate would be uploaded to the OTM system by each individual chooser after the utilities for the other candidates had presumably already been submitted, and the results had already been computed. The OTM system would then recompute the individual thresholds including the added candidate's utility ratings and the final social choice results would then be recomputed. The individual choosers would not have an incentive to rate the added candidate insincerely knowing that the OTM system would give them the strategically best outcome based on the complete list of submitted utilities. Therefore, candidate add-ons would not incentivize any individual chooser to choose insincerely. Furthermore, compliance with IIA is satisfied for add-ons since ratings for two candidates at a time could be uploaded for each individual chooser with thresholds recomputed at each step or as a final step thus demonstrating that the social choice can be arrived at by pairwise comparisons which Arrow's IIA demands.

Condition 4: The Social Welfare Function Is Not imposed.

The output of the OTM system is solely a function of the unrestricted inputs by assumption. There are no alternatives x and y such that xRy regardless of voter inputs. The OTM system is neutral and anonymous. It treats all citizens and alternatives the same. All permutations of V, the set of voters, and C, the set of candidates, are allowed. Permutations of voters or candidates do not change the results. The OTM mechanism, R, is neutral if it treats all the alternatives the same. R is neutral if for every permutation, ψ, of the set of voters, V, $R[\psi(v_1), ..., \psi(v_n)] = \psi[R(v_1, ..., v_n)]$.

Condition 5: The Social Welfare Function Is Not To Be Dictorial

For the OTM system $C_j = \{c_{1j}, c_{2j}, ... c_{nj}\}$. All permutations of c_{ij} are allowed, ∀ i,j. Condition (5) is satisfied since the winning set is based only on individual inputs which are all treated equally. There is no voter/consumer *j* such that xRy iff xR_jy. Therefore, the OTM mechanism satisfies all five of arrow's rational and normative conditions. Q.E.D.

Appendix B

We first define the following sets:

i) $V = \{v_1, v_2, \dots v_q\}$ is a set of choosers of size q, where $v_j \in V$ denotes the j^{th} chooser.

ii) $C = \{c_1, c_2, \dots, c_n\}$ is an ordered set of candidates of size n; candidates appear on the ballot in $c_1, c_2, \dots, c_n$ order. $c_i \in C$ denotes the i^{th} candidate.

iii) $C_j = \{c_{1j}, c_{2j}, \dots c_{nj}\}$ is the ordered set of preferences for alternatives of the j^{th} voter. $c_{1j} \mathrm{R_j} c_{2j} \mathrm{R_j} c_{3j} \mathrm{R_j} c_{4j}, \dots, c_{n-1j} \mathrm{R_j} c_{nj}$

iv) $U_j = \{u_{1j}, u_{2j}, \dots u_{nj}\}$ is a set of utilities of size n, with $u_{1j} \geq u_{2j} \geq \dots \geq u_{nj}$ and $0 \leq u_{ij} \leq 1$, $\forall$ i,j. U_j is the utility set of the j^{th} voter after applying an affine linear transformation to their submitted set of utilities such that $0 \leq u_{ij} \leq 1$. u_{ij} is the utility of candidate c_{ij}

v) $T_j = \{t_{1j}, t_{2j}, \dots t_{nj}\}$ is a set of thresholds of size n such that $t_{1j} \geq t_{2j} \geq \dots \geq t_{nj}$ and $0 \leq t_{ij} \leq 1$, $\forall\, i, j$.

vi) $X = \{x_1, x_2, \dots x_n\}$ $x_i = \{\mathbb{N}^0\}$, is a set of non-negative integers. X represents the cumulative votes for candidates as they appear on the ballot.

vii) $Y = \{y_1, y_2, \dots y_n\}$ is the set which orders the candidates by the number of votes received by each candidate. $y_1 R y_2 R \dots R y_n$.

viii) $W = \{w_1, w_2, ..., w_m\}$ is a set of candidates of size $m < n$ representing the unordered winning set.

ix) $B_j = \{b_{1j}, b_{2j}, ..., ,b_{nj}\}$ is a set of approval style votes in order of the j^{th} voter's candidate preferences. $b_{ij} = \{ \mathbb{N}^0 \mid 0, 1\}$

x) $O_j = \{o_{1j}, o_{2j}, ...\}$ O_j is the set of candidates given approval style votes of "1" by voter j, called the optimal set.

xi) $U_{aj} = \{u_{a1j}, u_{a2j}, ..., u_{anj}\}$ is the set of utilities above threshold for each chooser. u_{aij} is defined as the sum of utilities above threshold t_{ij} for voter j, $\forall$ i,j . The sum of utilities above threshold is computed for each of the n thresholds. n_{aij} is the corresponding number of utilities above threshold $\forall\ i, j$. u_{aij}/n_{aij} is the average utility above threshold.

We now define following functions:

i) $\tau : C \rightarrow X$ defines an ordered pair, (c_i, x_i) such that $\tau(c_i) = x_i$,the cumulative number of votes for each candidate.

ii) $\alpha : X \rightarrow Y$ α defines an ordered pair (x_r, y_r) such that $[\ y_r \mathrm{R} y_z$ iff $x_r \geq x_z\ ]$

for $1 \leq r, z \leq n$; *r , z, n integers.*

iii) $\beta : Y \rightarrow W$ such that $\beta(y_i) = w_i$ for $1 \leq i \leq m$. The function, β, places the top m vote getters in the winning set. If y_m represents a tie with y_{m+z} for $z \geq 1$, ties are resolved randomly so that W is always of size m.

iv) $\chi_j : C \rightarrow C_j$ The function χ_j assigns to each element $c_i \in C$ an element $\chi_j(c_i) = c_{ij}$ such that $c_{1j} R_j c_{2j} \ldots c_{(n-1)j} R_j c_{nj}$ for $1 \leq j \leq q$ where R_j means "is preferred or indifferent to". Each voter, j, orders the set of alternatives according to their preferences.

v) $\eta_j : C_j \rightarrow U_j$ the function η_j assigns to each element $c_{ij} \in C_j$ an element $\eta_j(c_{ij}) = u_{ij}$ where u_{ij} is the utility that is assigned to candidate c_{ij} by voter j.

vi) $\delta_j : C_j \rightarrow B_j$ defines an ordered pair (c_{ij}, b_{ij}) such that $\delta_j(c_{ij}) = b_{ij}$ for $1 \leq j \leq q$ and $1 \leq i \leq n$

vii) $\gamma_j : U_j \rightarrow T_j$ defines the relationship $\gamma_j(u_{ij}) = t_{ij}$ such that $t_{ij} = u_{ij} - \varepsilon$ where $\varepsilon \ll 1, \ \forall\, i, j$

viii) $\phi_{aj} : T_j \rightarrow U_{aj}$ such that $\phi_{aj}(t_{ij}) = u_{aij}$, where $u_{aij} = \sum_{u_{ij} > t_{ij}} u_{ij} \quad \forall\, i, j$

Appendix C

Examples

We have computed the expected average utility vs threshold for individual utility profiles U1 and U2 (dropping the j). We have plotted $E_{t_{ij}}(u_{W_j})$ (simplifying notation to E(u) vs threshold T) for $n = 21$, $1 \le i \le 21$, $m = 1 - 4$ as shown in Figures 2 and 3. Figure 2 represents a "smooth transition" between utilities. Figure 3 represents an "abrupt transition" between utilities.

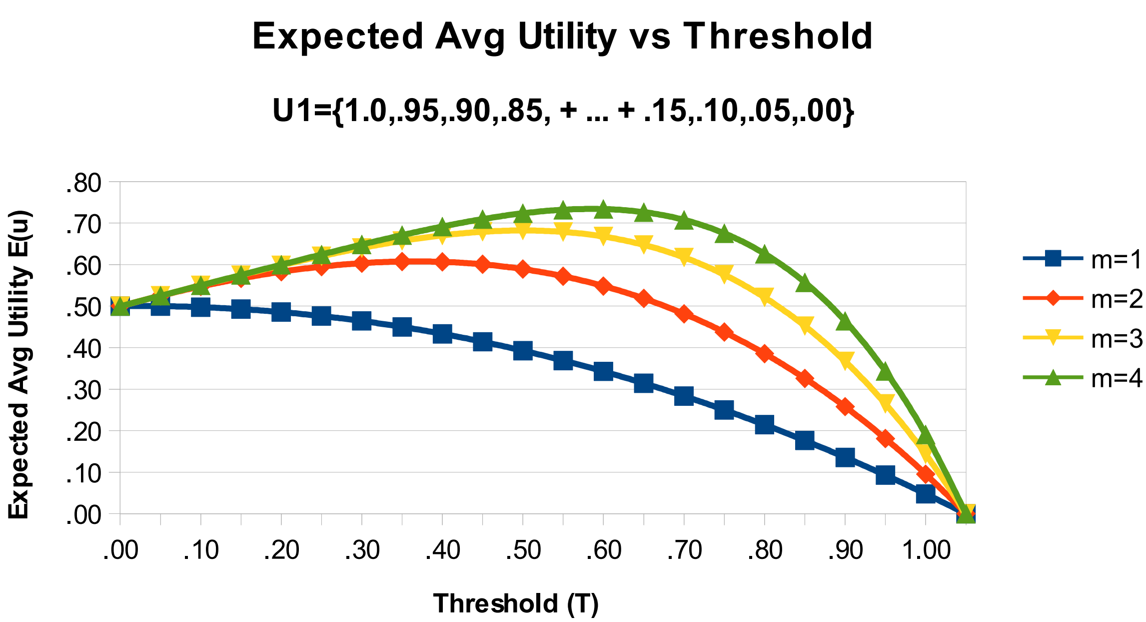

Figure 2

For m=1, E(u) max = .5000 @ T = 0.05. For m=2, E(u) max = .6075 @ T = 0.35. For m=3, E(u) max = .6823 @ T = 0.50. For m=4, E(u) max =.7338 @ T = 0.60

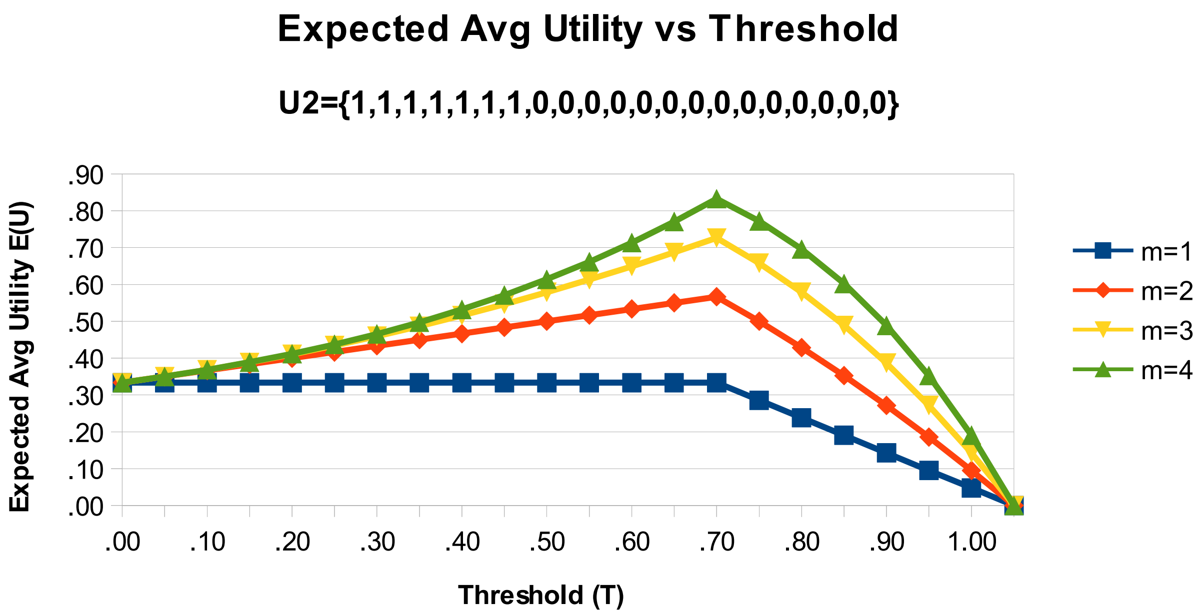

Figure 3

For m=1, E(u) max = 0.3333 @ T = 0.7. For m=2, E(u) max = 0.5667 @ T = 0.7. For m=3, E(u) max = 0.7263 @ T = 0.7. For m=4, E(u) max = 0.8327 @ T = 0.7.

We consider t_j^* to be the greatest value of T such that *E(u)* is a maximum i.e. limsup[*E(u)*] for 0 < T < 1, as shown in Figure 3 for *m* = 1. Figure 3 shows that the optimal threshold for this utility profile is always at 0.7 regardless of the value of *m* which is intuitively plausible. As *m* increases, the expected average utility of the social choice for an individual with this profile approaches +1.

Figure 2 shows that for utility profile U1 and *m* = 1 the best strategy is to give an approval style vote of "1" to all candidates except the one whose utility is "0". That one gets an approval style vote of "0". As the size of the winning set increases, however, fewer candidates are assigned an approval style vote of "+1", and the expected average utility of the winning set for the voter with this utility profile increases.

With regard to approval voting, Smith (2005) proves the following: “Mean-based thresholding is optimal range-voting strategy in the limit of a large number of other voters, each random independent full-range.” Range voting is similar to utilitarian voting. While Smith's analysis assumes a completely randomized set of utility profiles, it does not give the optimal strategy for any particular utility profile. Lehtinen (2010: pp. 285-310) has also used expected utility maximizing voting behavior to indicate which candidates should be given an approval style vote. He agrees with Smith that an approval style vote of "+1" should be given to all candidates for whom their utility exceeds the average utility of all candidates and a "0" otherwise. Brams and Fishburn (1983: p. 90) also agree with Smith and Lehtinen: "When cardinal utilities are associated with the preferences of a voter, his utility-maximizing strategy in large electorates is to vote for all candidates whose utilities exceed his average utility over all the candidates." These writers consider only single member districts.

Based on the examples in Figues 2 and 3 we would disagree. With regard to Figure 2, the average utility for a voter with profile U1 is 0.5, but our results show a maximum expected average utility at a threshold of 0.05 for $m = 1$, and progressively higher optimal thresholds for higher values of m. For Figure 3 the optimal threshold is 0.7 for all values of m with maximum expected average utility increasing as m increases. The average utility for U2 is 7/21 = 0.33. If the threshold for "+1" approval votes with $m = 1$ were to be set to 0.33 as the above writers suggest, the expected average utility would be the same as what it is at the optimal threshold of 0.7, but more candidates with utility values of zero would be given approval votes of "1".

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